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4 SEM TDC GEMT (CBCS) 4.1/4.2/4.3

2024

(May/June)

MATHEMATICS

(Generic Elective)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

All symbols have their usual meanings

Paper : GE—4.1

(Algebra)

UNIT—1

1. *(a)* State True or False : 1

Addition of natural numbers in binary composition is not associative.

(b) Find the elements of U(20). 1

(c) Show that the subset $\{1, -1, i, -i\}$ of the complex numbers is an Abelian group under complex multiplication. 5

(Turn Over)

(d) Prove that in a group G, $(ab)^{-1} = b^{-1}a^{-1}$, for all $a, b \in G$. 3

(e) Prove that a group in which every element is its own inverse is Abelian. 4

2. (a) Define quaternion group. 1

(b) Describe the symmetries of an isosceles triangle. 3

(c) Prove that the set of matrices

$$A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

where α is a real number, forms a group under matrix multiplication. 5

Or

Prove that every permutation of a finite set can be written as a product of disjoint cycles.

(d) Prove that the order of a cyclic group is equal to the order of its generator. 5

Or

Show that {1, 2, 3} under multiplication modulo 4 is not a group but that {1, 2, 3, 4} under multiplication modulo 5 is a group.

UNIT—2

3. (a) State True or False : 1

Order of a cyclic group is not equal to the order of its generator.

(b) Write all the left cosets of H in G if $G = S_3$ and $H = \{1, (13)\}$. 3

(c) Prove that a non-empty subset H of a group G is a subgroup of G iff $a, b \in H \Rightarrow ab^{-1} \in H$. 5

(d) Show that the centre of a group G is a subgroup of G. 5

Or

Prove that a subgroup of a cyclic group is cyclic.

4. (a) Prove that every subgroup of an Abelian group is normal. 2

(b) State and prove Lagrange's theorem. 4

(c) Prove that every quotient group of a cyclic group is cyclic. 4

(d) Prove that if p is a prime number, then any group G of order $2p$ has a normal subgroup of order p. 4

Or

Let G be a group and let H be a normal subgroup of G. Prove that the set $G/H = \{ah : a \in G\}$ is a group under the operation $(aH)(bH) = abH$.

UNIT—3

5. (a) Define zero divisor. 1

(b) State True or False : 1
A commutative ring R is called an integral domain if R has zero divisor.

(c) Prove that the set $R = \{0, 1, 2, 3, 4, 5\}$ is a commutative ring with respect to addition and multiplication modulo 6 as the two-ring composition. 5

(d) Prove that a ring R is without zero divisor if and only if the cancellation laws hold in R. 5

Or

Prove that every field is an integral domain.

6. (a) Define division ring. 1

(b) Show that a ring of order p^2 (p is a prime) may not be commutative. 3

(c) Show that the set of numbers of the form $a + b\sqrt{2}$, with a and b as rational numbers, is a field. 4

(d) Show that, a non-empty subset S of a ring R is a subring of R iff $x, y \in S \Rightarrow xy, x - y \in S$. 4

Or

Show that a commutative ring with unity is a field if it has no proper ideal.