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6 SEM TDC DSE MTH (CBCS) 2 (H)

6 SEM 2024 MATHEMATICS

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(May)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE-2

(Linear Programming)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

(a) Define slack and surplus variables in a linear programming problem. 2

(b) Solve by simplex method

$$\text{Min } Z = x_1 - 3x_2 + 2x_3$$

subject to

$$3x_1 - x_2 + 3x_3 \le 7$$
$$-2x_1 + 4x_2 \le 12$$
$$-4x_1 + 3x_2 + 8x_3 \le 10$$
$$\text{and} \quad x_1,\, x_2,\, x_3 \ge 0$$

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Or

Explain the various steps of the simplex method involved in the computation of an optimum solution to a linear programming problem. 7

(c) Answer any *two* : 8×2=16

(i) Solve the following linear programming problem by two-phase simplex method :

$$\text{Min } Z = x_1 + x_2$$

subject to

$$2x_1 + x_2 \geq 4$$
$$x_1 + 7x_2 \geq 7$$
$$\text{and} \quad x_1, x_2 \geq 0$$

(ii) Use Big-M method to solve the following Linear Programming Problem :

$$\text{Max } Z = x_1 + 2x_2 + 3x_3 - x_4$$

subject to

$$x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$
$$x_1 + 2x_2 + x_3 + x_4 = 10$$
$$\text{and} \quad x_1, x_2, x_3, x_4 \geq 0$$

(iii) Solve the Linear Programming Problem :

$$\text{Min } Z = 5x_1 + 3x_2$$

subject to

$$2x_1 + 4x_2 \leq 12$$
$$2x_1 + 2x_2 = 10$$
$$5x_1 + 2x_2 \geq 10$$
$$\text{where} \quad x_1, x_2 \geq 0$$

by Big-M method.

2. (a) If the objective of the primal is to maximize, then write the objective of the dual. 1

(b) Write the dual of the following linear programming problem :

$$\text{Max } Z = x_1 - x_2 + 3x_3$$

subject to

$$x_1 + x_2 + x_3 \leq 10$$
$$2x_1 - x_2 - x_3 \leq 2$$
$$2x_1 - 2x_2 - 3x_3 \leq 6$$
$$\text{and} \quad x_1, x_2, x_3 \geq 0$$

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(c) Answer any *two* from the following : 5×2=10

(i) Write the mathematical formulation of the dual linear programming problem in symmetrical form.

(ii) Prove that the dual of a dual is primal.

(iii) Give an economic interpretation of dual variables.

3. *(a)* Write the necessary and sufficient condition for a feasible solution to a transportation problem. 2

(b) Write the conditions for a non-degenerate basic feasible solution. 2

(c) Answer any *two* : 8×2=16

(i) Describe the computational procedure of the MODI method in a transportation problem.

(ii) Find the initial basic feasible solution using Vogel's Approximation method and find the optimal solution :

	D_1	D_2	D_3	D_4	*Supply*
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	

(iii) A department of a company has five employees with five jobs to be performed. The time (in hours) that each man takes to perform each job is given in the effectiveness matrix :

Employees

Jobs	*I*	*II*	*III*	*IV*	*V*
A	10	5	13	15	16
B	3	9	18	13	6
C	10	7	2	2	2
D	7	11	9	7	12
E	7	9	10	4	12

How should the jobs be allocated, one per employee, so as to minimize the total man hours?

4. *(a)* What is a strictly determine game in game theory? 1

(b) Answer any *two* : 5×2=10

(i) Solve the following game stating the optimal strategies and the saddle point : 5

$$\begin{bmatrix} 2 & 3 & 2 & 4 & 6 \\ 0 & -2 & 1 & 2 & 1 \\ -1 & 3 & 0 & -1 & 3 \\ 4 & 5 & -1 & 2 & 1 \\ 3 & 2 & -2 & 1 & -2 \end{bmatrix}$$

(ii) Find the value of the 2×2 game algebraically by using mixed strategies : 5

Player A

	B_1	B_2
Player B A_1	2	3
A_2	4	−1

(iii) Solve the following 2×4 game geometrically : 5

Player B

	B_1	B_2	B_3	B_4
Player A A_1	3	2	−1	4
A_2	2	5	6	−2

(c) Solve the game problem by using LP method : 9

Player B

	B_1	B_2	B_3
Player A A_1	1	0	−2
A_2	0	3	2

Or

State the modified dominance property. Reduce the following game to 2×2 game by using dominance and modified dominance property and then solve the game :

Player B

	B_1	B_2	B_3	B_4
Player A A_1	1	2	−2	2
A_2	3	1	2	3
A_3	−1	3	2	1
A_4	−2	2	0	−3

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