Total No. of Printed Pages—9

3 SEM TDC STSH (CBCS) C 5 (N/O)

2023

(Nov/Dec)

STATISTICS

(Core)

Paper : C–5

(Sampling Distribution)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the following : 1×5=5

(a) If X is a continuous random variable with mean μ and variance σ^2, then for any positive number K

$$P\{|X-\mu| \geq K\sigma\} \leq \frac{1}{K^2}$$

is known as

(i) Chebychev's inequality

(ii) weak law of large number

(iii) strong law of large number

(iv) None of the above

(b) The SE of mean of a random sample of size n from a population with variance 4 is

(i) 2

(ii) $\frac{2}{n}$

(iii) $\frac{2}{\sqrt{n}}$

(iv) $\frac{n}{2}$

(c) The range of χ^2-variate is

(i) $-\infty$ to ∞

(ii) 0 to ∞

(iii) 0 to 1

(iv) $-\infty$ to 0

(d) The area of critical region depends on

(i) the size of type–I error

(ii) the size of type–II error

(iii) the value of the statistic

(iv) the number of observations

(e) The variance of Student's t-distribution is

(i) n

(ii) $2n$

(iii) $\frac{\nu}{\nu-2}$, where ν = degrees of freedom

(iv) None of the above

2. Answer the following questions : 2×5=10

(a) Define convergence in probability.

(b) What are type–I error and type–II error?

(c) What are the uses of order statistics?

(d) Define F-statistic. Write down its probability density function.

(e) State additive property of χ^2-variates.

3. (a) Obtain the sampling distribution of mean of a random sample drawn from a normal population with mean μ and variance σ^2. 6

Or

(b) Define parameter and statistic. Explain the importance of sampling distribution of a statistic in Statistics. 2+4=6

4. Describe, how you would test the difference of two means in large samples. 4

5. (a) Define rth order statistic $X_{(r)}$. Obtain the joint probability density function of $X_{(r)}$ and $X_{(s)}$, $r < s$ in a random sample of size n from a population with continuous distribution function $F(x)$. Hence deduce the probability density function of sample range $W = X_{(n)} - X_{(1)}$. 7

Or

(b) State Chebychev's inequality. If X is the number scored in a throw of a fair die, show that the Chebychev's inequality gives $P\{|X - \mu| > 2 \cdot 5\} < 0 \cdot 47$, where μ is the mean of X, while the actual probability is zero. 2+5=7

6. (a) Define χ^2-statistic. Derive the probability density function of χ^2 with n degrees of freedom using moment-generating function. 6

Or

(b) Show that χ^2-distribution approaches to normality when $n \to \infty$.

7. (a) Describe the χ^2-test of goodness of fit. 5

Or

(b) Find the mode of χ^2-distribution with n degrees of freedom.

8. Answer any *two* of the following questions : 6×2=12

(a) Write down the density function of t-distribution. Describe the chief features of t-distribution curve.

(b) Derive the distribution of F.

(c) Find the variance of the t-distribution with n degrees of freedom ($n > 2$).

(d) Obtain the mode of F-distribution with (n_1, n_2) degrees of freedom and show that it lies between 0 and 1.

(Old Course)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

1. Choose the correct answer from the following : 1×5=5

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is known as

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