3 SEM TDC PHYH (CBCS) C 5

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(Nov/Dec)

PHYSICS

(Core)

Paper : C–5

(Mathematical Physics—II)

$$\frac{Full\ Marks : 53}{Pass\ Marks : 21}$$

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Choose the correct answer :　　　　　1×5=5

   (a) The value of erf (∞) is

   　　(i) 1

   　　(ii) 0

   　　(iii) −1

   　　(iv) None of the above

(b) The value of $\Gamma\left(-\frac{1}{2}\right)$ is

    (i) $\sqrt{\pi}$

    (ii) $\dfrac{-\pi}{2}$

    (iii) $-2\sqrt{\pi}$

    (iv) 0

(c) The value of $\int_{-1}^{+1}[P_3(x)]^2\,dx$ is

    (i) $\dfrac{2}{3}$

    (ii) $\dfrac{2}{7}$

    (iii) $\dfrac{1}{7}$

    (iv) None of the above

(d) The value of Hermite polynomial $H_2(x)$ is

    (i) $\dfrac{1}{2}(3x^2-1)$

    (ii) $\dfrac{1}{3}(3x^2-1)$

    (iii) $(4x^2-2)$

    (iv) $\dfrac{1}{2}(4x^2-1)$

(e) According to the Parseval's formula for Fourier series, the integral $\int_{-c}^{c}[f(x)]^2\,dx$ equals to

    (i) $c\left\{a_0^2 + \sum\limits_{n=1}^{\infty}(a_n^2 + b_n^2)\right\}$

    (ii) $c\left\{\dfrac{1}{2}a_0^2 + \sum\limits_{n=1}^{\infty}(a_n^2 + b_n^2)\right\}$

    (iii) $c\left\{a_0^2 + \dfrac{1}{2}\sum\limits_{n=1}^{\infty}(a_n^2 + b_n^2)\right\}$

    (iv) None of the above

2. (a) Describe Dirichlet's conditions for a Fourier series.   2

   (b) Expand the function $f(x)$ in a Fourier series where $-\pi < x < \pi$ and $f(x)$ is given by

$$f(x) = 0 \quad \text{when} \quad -\pi < x \leq 0$$
$$= \frac{\pi x}{4} \quad \text{when} \quad 0 < x \leq \pi$$

Hence show that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

    4+2=6

(c) Obtain the complex form of the Fourier series for the function defined as follows :   3

$$f(x) = 0 \quad \text{when} \quad -\pi < x \leq 0$$
$$= 1 \quad \text{when} \quad 0 < x \leq \pi$$

3. (a) Determine the nature of the point $x = 0$ for $x\dfrac{d^2y}{dx^2} + y\sin x = 0$.   1+2=3

(b) Solve using Frobenius method (any *one*) :   5

    (i) $x^2 y'' + (x + x^2)y' + (x - 9)y = 0$

    (ii) $4xy'' + 2y' + y = 0$

(c) Express the following in terms of Legendre polynomials :   3

$$4x^2 - 3x + 2$$

(d) Prove that

$$\int_{-1}^{+1} x P_n(x) P_{n-1}(x)\, dx = \frac{2n}{4n^2 - 1}$$ 4

Or

Prove that

$$\int_{-\infty}^{\infty} e^{-x^2} H_m(x) H_n(x) = 0 \quad \text{for } m \neq n$$

(Continued)

4. Evaluate :

$$\int_0^\infty x^{n-1} e^{-h^2 x^2}\, dx = \frac{1}{2h^n}\Gamma\left(\frac{n}{2}\right)$$ 3

Or

Prove that

$$\int_0^{\pi/2} \sin^p \theta \sin^q \theta\, d\theta = \frac{\Gamma\left(\dfrac{p+1}{2}\right)\Gamma\left(\dfrac{q+1}{2}\right)}{2\Gamma\left(\dfrac{p+q+2}{2}\right)}$$

5. Answer any *one* of the following :   6

(a) What is relative error? Describe with an example. $R = \dfrac{4xy^2}{z^3}$ and errors in x, y, z be 0·001, show that the maximum relative error at $x = y = z = 1$ is 0·006.   1+1+4=6

(b) Describe briefly the least square method of curve fitting. Fit a curve to the following data by the least square method :   2+4=6

x	1	2	3	4
y	1·7	1·8	2·3	3·2

(Turn Over)

6. (a) Solve any *two* of the following partial differential equations by method of separation of variables : 4×2=8

(i) $\dfrac{\partial^2 z}{\partial x^2} - 2\dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y} = 0$

(ii) $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$ if $u(x, 0) = \dfrac{1}{2}x(1-x)$

(iii) $\dfrac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$ given that :

$$u = 0 \text{ when } t = 0$$

$$\dfrac{\partial u}{\partial t} = 0 \text{ when } x = 0$$

(b) Find the solution of 2-D Laplace's equation in cylindrical polar co-ordinates. 5

Or

Find the vibration modes of a stretched string by solving the one-dimensional wave equation.

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