5 SEM TDC DSE MTH (CBCS) 1.1/1.2/1.3 (H)

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(November)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–1

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

Paper : DSE–1.1

(Analytical Geometry)

1. Answer the following questions :

   (a) Write the centre of the conic

   $$\frac{(x-2)^2}{9} + \frac{(y+1)^2}{16} = 1$$

   1

   (b) Write the processes to sketch the hyperbola.

   4

(*Turn Over*)

(c) Find the centre, foci, vertices and length of the major axis of the conic

$$4x^2 + y^2 + 8x - 2y + 1 = 0$$

4

(d) Describe the graph of the curve

$$16x^2 + 9y^2 - 64x - 54y + 1 = 0$$

Also find its centre and foci.

6

Or

Describe the graph of the hyperbola

$$x^2 - y^2 - 4x + 8y - 21 = 0$$

and sketch its graph.

2. Answer the following questions :

(a) Write the condition of tangency of the line $y = mx + c$ to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. 1

(b) Write True or False :

1

A parabola is the set of all points in the plane that are equidistant from a fixed line and a fixed point not on the line.

(c) Define an ellipse.

1

(d) Find the equation of the ellipse, one of whose foci is $(-1, 1)$, eccentricity is $\dfrac{1}{2}$ and the corresponding directrix is $y = x + 3$. 5

(e) Find the equation of the parabola whose axis is parallel to the y-axis that has its vertex at $(5, -2)$ and passes through the point $(9, 5)$. Also sketch it. 7

Or

Find and sketch the curve of the hyperbola whose foci $(6, 4)$ and $(-4, -4)$ and eccentricity is 2.

3. Answer the following questions :

(a) Write the condition that the quadratic equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

represents parabola.

1

(b) Determine a rotation angle θ that will eliminate the xy-term of the conic

$$2x^2 + \sqrt{3}xy + y^2 - 10 = 0$$

2

(c) Consider the equation

$$x^2 - 4xy - 2y^2 - 6 = 0$$

Rotate the coordinate axes to remove the xy-term, then identify the type of conic represented by the equation and sketch its graph.

6

(4)

(d) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle $\theta = 45°$. Find an equation of the curve $3(x')^2 + (y')^2 = 6$ in xy-coordinate.   6

Or

Identify and sketch the curve

$$52x^2 - 72xy + 73y^2 + 40x + 30y - 75 = 0$$

4. Answer the following questions :

(a) Write the general equation of sphere.   1

(b) Write True or False :   1

The section of a sphere by a plane is a sphere.

(c) Write the standard equation of hyperbola of two sheets.   1

(d) Find the centre and radius of the sphere

$$x^2 + y^2 + z^2 - 8x + 4y - 6z + 4 = 0$$ 2

(e) A plane passes through a fixed point (a, b, c) and meets the axes A, B, C. Show that the locus of the sphere $OABC$ is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$, where O is the origin.   5

(5)

(f) Find the equation of the sphere through the origin and intersecting coordinate axes at distances a, b, c from the origin.   5

Or

Obtain the equation of the sphere circumscribing the tetrahedron by the plane $x = y = z = 0$ and $2x + 3y + 4z - 12 = 0$.

5. Answer the following questions :

(a) Find the radius and the centre of the circle

$$x^2 + y^2 + z^2 + 12x - 13y - 16z + 111 = 0,$$
$$2x + 2y + z = 17$$ 5

(b) Find the equation of the sphere for which the circle

$$x^2 + y^2 + z^2 + 7y - 2z + 2 = 0,$$
$$2x + 3y - 4z = 8$$

is a great circle.   5

Or

Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere

$$x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0,$$

also find their point of contact.

6. Answer the following questions :

(a) Find the condition that the plane $lx + my + nz = p$ may touch the sphere

$$x^2 + y^2 + z^2 + 2gx + 2fy + 2hx + d = 0 \qquad 5$$

(b) Classify and sketch the quadric surface (any *one*) : $\qquad$ 5

(i) $\dfrac{x^2}{9} + \dfrac{y^2}{4} - \dfrac{z^2}{16} = 1$

(ii) $z = \dfrac{x^2}{8} + \dfrac{y^2}{2}$

Paper : DSE–1.2

(Portfolio Optimization)

1. Answer any *five* of the following questions : 1×5=5

(a) What is risk averse?

(b) What is portfolio?

(c) What is the value of variance of risk-free investment?

(d) Define business risk.

(e) Define risk-free asset.

(f) What is mutual fund?

2. (a) If an investment that costs $300 and is worth $350 after being held for two years, find annual holding period return (annual HPR) and annual holding period yield (annual HPY). $\qquad$ 4

(b) Define expected return of an investment. Calculate the expected rate of return of the following economic scenarios : $\qquad$ 1+2=3

Economic Conditions	Probability	Rate of Return
Strong economy	0·25	0·20
Weak economy	0·25	–0·20
No major change in economy	0·50	0·10

(8)

(c) Write the measures of risk in terms of variance and standard deviation of the estimated distribution of expected returns. Define the relative measure 'coefficient of variation' of risk. 2+2+1=5

(d) Describe different types of risk of an investment. 5

Or

Write three ways to change the relationship between risk and the required rate of return for an investment.

(e) Describe the relationship between risk and return. 4

(f) Describe the investment objectives for 25 years old investors and 65 years old investors. 4

3. (a) State one-fund theorem. 2

(b) What are the assumptions of the Markowitz portfolio theory? 5

(9)

(c) Write the formula for the expected return for a portfolio of investments. Calculate the expected return of portfolio of risky assets given by the table : 1+2=3

Weight (w_j) (Percent of Portfolio)	Expected Security Returns (R_i)
0·20	0·10
0·25	0·11
0·25	0·12
0·30	0·13

(d) What are the variance and standard deviation of returns for an individual investment? Calculate the variance for an individual risky asset given by the following table : 2+2+3=7

Possible Rate of Return (R_j)	Expected Security Return [$E(R_j)$]	Probabilities (P_i)
0·08	0·103	0·35
0·10	0·103	0·30
0·12	0·103	0·20
0·14	0·103	0·15

Or

Describe variance and standard variation of returns for a portfolio of investments. 7

(e) Define risk-free portfolio using standard deviation of a portfolio of investments. 2

(f) Write short notes on any *two* of the following : 3×2=6

   (i) Optimal portfolio

   (ii) Efficient frontier

   (iii) Changes of slope of SML

4. Answer any *three* of the following questions : 5×3=15

(a) Write five assumptions of capital market theory.

(b) Discuss the relations between risk and diversification of portfolio.

(c) Derive the equation of the capital asset pricing model (CAPM).

(d) Determine the expected rate of return with CAPM for the following five stocks :

Stock	Beta
A	0·70
B	1·00
C	1·15
D	1·40
E	−0·30

where, economy's $RER = 0.05$, and expected return on the market portfolio $E(R_M) = 0.09$.

5. What is security market line (SML)? Draw a rough diagram of SML showing low risk, average risk and high risk of an investment. 1+2=3

6. Suppose that during the most recent 10 years period, the average annual total rate of return including dividends on an aggregate market portfolio was 14 percent ($\overline{R}_M = 0.14$) and the standard deviation of annual rate of return for the market portfolio over past 10 years was 20 percent ($\sigma_M = 0.20$). Examine the risk-adjusted performance of the following portfolios using Sharpe measure :

Portfolio	Average Annual Rate of Return	Standard Deviation of Return
D	0·13	0·18
E	0·17	0·22
F	0·16	0·23

Also, plot their Sharpe measures with capital market line (CML). 7

Or

Describe Treynor portfolio performance measure with example.

Paper : DSE–1.3

(Financial Mathematics)

(For 2020 batch or later)

1. (a) Let the supply function of an item is given by $5q + 3p = 65$. Write the inverse supply function. 1

   (b) Let P be the selling price of an item. Write the revenue after introduction of excise tax T. 1

   (c) Let c be the capital after n years, where r is the rate of interest. Write the present value. 1

   (d) Let the demand and supply functions are given by $4q + p = 11$, $5q - 2p = 4$ respectively. Find the equilibrium set. 2

   (e) Find the solution of the recurrence equation $y_t = 5y_{t-1} + 6$. Given $y_0 = \dfrac{5}{3}$. 5

   Or

   Find the present value of an annuity of H for N years, at given fixed interest r.

(Continued)

2. (a) Write the economic interpretation of stable equilibrium. 2

   (b) For the supply and demand set
   $$S = \{(q, p) : q = bp - a\} \text{ and }$$
   $$D = \{(q, p) : q = c - dp\}$$
   Find the equilibrium price. 2

   (c) The supply and demand sets are given by $\{(q, p) : 3q - p = -16\}$, $\{(q, p) : q + p = 28\}$. Find the price sequence. 4

   Or

   Determine whether the Cobweb model predicts stable or unstable equilibrium for the market with $q^S(p) = 2p - 3$ and $q^D(p) = 18 - p$.

3. (a) Define marginal cost. 1

   (b) Let a firm has cost function
   $$C(q) = 700 + 21q - 3q^2 + q^3$$
   Show that its marginal cost is always positive. 2

(Turn Over)

(c) Let supply and demand functions are given by $3q - 2p = 12$ and $2q + 3p = 48$. An excise tax T per unit is imposed. Determine when the revenue is maximum. 5

Or

Find the maximum and minimum values of the function $f(x) = x^4 - 8x^3 + 16x^2 - 5$ in the interval $[1, 5]$.

4. (a) Write the mathematical representation of elasticity of demand. 1

(b) Define marginal revenue. 2

(c) Let for an efficient small firm, the cost function is $C(q) = q^3 - 10q^2 + 110q + 180$ and maximum production capacity per day is 12 units. Determine (i) profit function and (ii) break-even point. 2+2=4

(d) Discuss elasticity of the demand function. 5

Or

Let the demand set for an item is

$$D = \{(q, p) : q^2(2 + p^3) = 200\}$$

Determine the values of p where the demand is elastic.

(Continued)

5. (a) Define portfolio of an investor. 2

(b) Describe an economy with many industries. 4

(c) Find the extreme value of the function

$$f(x, y) = 3x^2 + 2xy + 2y^2 - 160x - 120y + 8$$ 6

Or

Let the demand for two items are

$$x = 2 - 2p^x + 3p^y \text{ and } y = 9 + 3p^x - p^y$$

The cost function

$$C(x, y) = 8 + x + 2x^2 - xy + y^2$$

Find the profit function.

6. (a) Define Leontief matrix. 2

(b) Describe return matrix. 2

(c) Describe how to make money with matrices with an example. 6

Or

Find the equation for production schedule $\bar{x} = (x_1, x_2)$ in terms of external demand $\bar{d} = (d_1, d_2)$, where input-output model with two industries is given by

$$A = \begin{bmatrix} 0 \cdot 5 & 0 \cdot 2 \\ 0 \cdot 3 & 0 \cdot 4 \end{bmatrix}$$

7. (a) Define derivative asset.   1

  (b) Write one form of hedging.   1

  (c) Write one use of comparison principle.   1

  (d) Write by which investments are described.   1

  (e) Define cash flow stream.   2

8. (a) Define nominal interest rate.   1

  (b) Define discount factor.   1

  (c) Let r be the rate of interest per year. Write the amount after one year, if p is the principal amount.   1

  (d) Write the continuous compounding formula for present value.   2

  (e) Describe compounding at various intervals.   4

Or

Let (100, 200, 300, 400) be the cash flow stream and the rate of interest is 10% for each period. Find the future value at the end of cash flow.

  (f) Find the internal rate of return of the cash flow (1, 0, −1, 1).   5

(*Continued*)

Paper : DSE–1.3

(Financial Mathematics)

(For 2019 batch only)

UNIT—I

1. Answer the following questions :   1×4=4

  (a) Define nominal interest rate.

  (b) Write one form of hedging.

  (c) Mention the comparison principle.

  (d) Define simple interest rate.

2. Answer the following questions :   2×4=8

  (a) Define cash flow stream.

  (b) Write the continuous compounding formula for present value.

  (c) Write the nature of growth under compound interest.

  (d) Write about the time value of money.

(*Turn Over*)

3. Answer any *four* of the following questions :

$$6 \times 4 = 24$$

(a) Find the internal rate of return of the cash flow $(1, 0, 1, -1)$.

(b) Describe compounding at various levels.

(c) Find the present value of a cash flow stream.

(d) Find the corresponding effective rate for 3% compounded monthly.

(e) Derive the mathematical formula to calculate annuity.

4. Describe mortgages. 4

UNIT—II

5. Answer the following questions :

$$1 \times 4 = 4$$

(a) Write for what purpose net present value is used.

(b) Define money market.

(c) Write when security is termed as zero coupon bond.

(d) Write True or False :

Annuities are traded.

6. Answer the following questions : $2 \times 4 = 8$

(a) Write the formula to calculate annuity.

(b) Define amortization.

(c) Write about face value of a bond.

(d) Describe price-yield curve.

7. Answer any *two* of the following questions :

$$4 \times 2 = 8$$

(a) Compute future value of cash flow stream $(-2, 2, 1.5, 1)$, the periods are years and interest rate is 10%.

(b) Describe maturity.

(c) Describe duration.

8. Answer any *four* of the following questions :

$$5 \times 4 = 20$$

(a) Describe municipal bonds and corporate bonds.

(b) Derive the mathematical formulation of Macaulay duration.

(**20**)

(c) Describe immunization.

(d) An 8% bond with 15 years to maturity has a yield of 9%. Determine the price of this bond.

(e) Describe the derivation of variance of the rate of return of the portfolio.

(f) Describe two-fund theorem.

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