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4 SEM TDC STSH (CBCS) C 9

2023

(May/June)

STATISTICS

(Core)

Paper : C–9

(Linear Models)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the given alternatives in each question : 1×5=5

(a) In the linear model, $y = X\beta + \varepsilon$, the Gauss-Markov setup can be expressed as

(i) $(y, X\beta, \sigma^2 I)$

(ii) $E(y) = X\beta$, $\text{cov}(y) = \sigma^2\hat{\beta}$

(iii) $E(\varepsilon) = 0 \; \forall X$, $\text{cov}(\varepsilon) = \sigma^2 u$

(iv) All of the above

(b) With usual notation for multiple linear regression model, estimate of σ^2 is given by

(i) $\frac{y'y - \hat{\beta}'X'y}{n-k-1}$

(ii) $\frac{y'y - \hat{\beta}'X^{-1}y}{n-k-1}$

(iii) $\frac{y'y - \hat{\beta}'X'y}{n-k}$

(iv) None of the above

(c) ANOVA technique is designed to

(i) compare the homogeneity of several means simultaneously

(ii) test the equality of several population variances

(iii) Both (i) and (ii)

(iv) Neither (i) nor (ii)

(d) For ANOVA technique, the concomitant variable

(i) may be a quality characteristic

(ii) is not necessarily be measurable

(iii) is a quality characteristic, which can be converted to numerical score

(iv) All of the above

(e) The assumption of constant variance of the residual is known as

(i) autocorrelation

(ii) heteroscedasticity

(iii) homoscedasticity

(iv) multicollinearity

2. Answer in brief of the following : 2×5=10

(a) Write two properties of estimability of linear parametric function.

(b) Write the multiple linear regression model and explain the terms. Write the assumptions to fit the model.

(c) Define linear model in ANOVA setup.

(d) What do you mean by concomitant variable?

(e) Mention any two methods which are commonly used as model adequacy checking.

3. (a) Estimate the error variance or residual sum of squares of the general linear model in matrix form. 7

Or

(b) Establish the mean and variance of $\hat{\beta}$, the estimated parameter vector of the general linear model in matrix form. 3½+3½=7

4. (a) What are the assumptions of the simple linear regression model? For a simple linear regression model, $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$, find the $100(1-\alpha)\%$ confidence interval for the mean response at a particular value of the regressor variable X. 2+5=7

Or

(b) For multiple linear regression model, with usual notation in matrix form, if $E(y) = X\beta$ and $\text{cov}(y) = \sigma^2 I$, the least square estimators $\hat{\beta}$ have minimum variance among all linear unbiased estimators. 7

5. Answer any *two* of the following : 7×2=14

(a) Write short notes on fixed effect and random effect models.

(b) Explain the meaning of analysis of variances and give its applications. What is the difference between 'variability within classes' and 'variability between classes'?

(c) Describe analysis of variance of one-way classification of fixed effect model, stating clearly—

(i) the hypothesis to be tested;

(ii) the assumptions to be made;

(iii) the least squares estimates of the parameter;

(iv) the partitioning of sum of squares with d.f.;

(v) the test statistics to be used;

(vi) the ANOVA table.

(d) Outline the various steps in carrying out the ANOVA of a two-way classified data with one observation per cell, stating clearly—

(i) the fixed effects mathematical model;

(ii) the assumptions used;

(iii) the hypothesis to be tested;

(iv) the partitioning of various sum of squares and d.f.;

(v) the ANOVA table.

(e) Give the fixed effect mathematical model for two-way classification with one observation per cell, stating clearly the assumptions involved. Also obtain—

(i) the estimates of the parameters in the model;

(ii) the variance of the estimates.

6. Write an explanatory note on the following : 7

(a) Model adequacy checking

Or

(b) Violation of the assumptions in the regression model

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