Total No. of Printed Pages—15

2 SEM TDC STSH (CBCS) C 4 (N/O)

2023

(May/June)

STATISTICS

(Core)

Paper : C–4

(Algebra)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the given alternatives in each question : 1×6=6

(a) A polynomial of three terms is called a

(i) monomial

(ii) binomial

(iii) trinomial

(iv) All of the above

(b) If $\begin{bmatrix} 5 & k+2 \\ k+1 & -2 \end{bmatrix} = \begin{bmatrix} k+3 & 4 \\ 3 & -k \end{bmatrix}$, then $k =$

(i) 0

(ii) 2

(iii) −2

(iv) 1

(c) Cofactor of 4 in the determinant

$$\begin{vmatrix} 1 & 2 & -3 \\ 4 & 5 & 0 \\ 2 & 0 & 1 \end{vmatrix}$$

is equal to

(i) 2

(ii) −2

(iii) −5

(iv) None of the above

(d) If one root of the equation

$$\begin{vmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix} = 7$$

is $x = -9$, then the other roots are

(i) (2, 6)

(ii) (3, 6)

(iii) (2, 7)

(iv) (3, 7)

(e) If a matrix A has a non-zero minor of order r, then

(i) $\rho(A) = r$

(ii) $\rho(A) \geq r$

(iii) $\rho(A) < r$

(iv) $\rho(A) \leq r$

(f) The matrix of a quadratic form is

(i) symmetric

(ii) anti-symmetric

(iii) orthogonal

(iv) Hermitian

2. Answer the following : 3×4=12

(a) Solve the equation

$$x^4 - 7x^3 + 27x^2 - 47x + 26 = 0$$

given that one of its root is $2 + 3i$.

(b) If $\begin{bmatrix} A^{-1} & 0 \\ X & A^{-1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ B & A \end{bmatrix}^{-1}$ and A is non-singular, then find out the value of X.

(c) Show that

$$\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ca \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

(d) Reduce the matrix A to its normal form, where

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

Hence find the rank of A.

3. Answer any *one* of the following : 6

(a) (i) If $(ax^3 + bx^2 + cx + d)$ be divisible by $(x^2 + l^2)$, then show that $ad = bc$.

(ii) Find the equation whose roots are 1, −2, 3, −4.

(b) (i) Solve the equation

$$x^3 - 9x^2 + 23x - 15 = 0$$

whose roots are in AP.

(ii) If the equation

$$x^4 + ax^3 + bx^2 + cx + d = 0$$

has three equal roots, then show that each of them is equal to

$$\frac{6c - ab}{3a^2 - 8b}$$

(c) (i) Prove that, if two vectors are linearly dependent, one of them is a scalar multiple of the other.

(ii) Define basis of a vector space.

4. Answer any *two* of the following : 3×2=6

(a) If A and B are two idempotent matrices, then show that $A + B$ will be idempotent if $AB = BA = 0$.

(b) Show that any diagonal element of a Hermitian matrix is necessarily real.

(c) If A and B are two n-rowed orthogonal matrices, then AB and BA are also orthogonal matrices.

(d) Prove that the necessary and sufficient condition for a square matrix A to possess the inverse is that $|A| \neq 0$.

(e) Let A and B be two square matrices of order n. Then prove that

$$\text{tr}(AB) = \text{tr}(BA)$$

5. Answer any *three* of the following : 5×3=15

(a) Write the general definition of determinant and mention its properties. 2+3=5

(b) Define adjoint of a square matrix. If $A_{n \times n}$ is a square matrix of order n, then prove that

$$A(\text{adj } A) = (\text{adj } A)A = |A| I_n$$ 2+3=5

(c) Solve the following equations by using Cramer's rule : 5

$$x+2y+3z=6$$
$$2x+4y+z=7$$
$$2x+2y+9z=14$$

(d) Define circulant determinant and Vandermonde determinant for *n*th order. 2½+2½=5

(e) Investigate for what values of λ and μ, the system of simultaneous equations

$$x+y+z=6$$
$$x-2y+3z=10$$
$$x+2y+\lambda z=\mu$$

has *(i)* no solution, *(ii)* a unique solution and *(iii)* an infinite number of solutions. 1½+1½+2=5

(f) Given that

$$A=\begin{bmatrix}1 & -2 & -1\\ 2 & 3 & 1\\ 0 & 5 & -2\end{bmatrix}$$

Compute *(i)* $|A|$, *(ii)* adj A and *(iii)* A^{-1}. 1+2+2=5

6. Answer any *two* of the following : 5×2=10

(a) Prove that rank of a non-singular matrix is equal to the rank of its reciprocal matrix.

(b) Under what condition the rank of the following matrix A is 3? Is it possible for the rank to be 1? Why?

$$A=\begin{bmatrix}2 & 4 & 2\\ 3 & 1 & 2\\ 1 & 0 & x\end{bmatrix}$$

(c) Obtain a *g*-inverse of the matrix given by

$$A=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix}$$

(d) Find the characteristic roots and characteristic vectors of the matrix

$$A=\begin{bmatrix}a & h & g\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$$

(e) State and prove Cayley-Hamilton theorem.

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. Choose the correct answer from the given alternatives in each question : 1×8=8

(a) A polynomial of three terms is called a

(i) monomial

(ii) binomial

(iii) trinomial

(iv) None of the above

(b) The fundamental theorem of algebra states that, every algebraic equation has

(i) at least one root, real or imaginary

(ii) only two roots, both are real

(iii) only two roots, one real and other imaginary

(iv) at least one root, which is real

(Continued)

(c) If $\begin{bmatrix} 5 & k+2 \\ k+1 & -2 \end{bmatrix} = \begin{bmatrix} k+3 & 4 \\ 3 & -k \end{bmatrix}$, then $k =$

(i) 0

(ii) 2

(iii) −2

(iv) 1

(d) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, then $A^{-1} =$

(i) $\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} -5 & 2 \\ -2 & 1 \end{bmatrix}$

(e) Cofactor of 4 in the determinant

$$\begin{vmatrix} 1 & 2 & -3 \\ 4 & 5 & 0 \\ 2 & 0 & 1 \end{vmatrix}$$

is equal to

(i) 2

(ii) −2

(iii) −5

(iv) None of the above

(Turn Over)

(f) If one root of the equation

$$\begin{vmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix} = 7$$

is $x = -9$, then the other roots are

(i) (2, 6)

(ii) (3, 6)

(iii) (2, 7)

(iv) (3, 7)

(g) If a matrix A has a non-zero minor of order r, then

(i) $\rho(A) = r$

(ii) $\rho(A) \geq r$

(iii) $\rho(A) < r$

(iv) $\rho(A) \leq r$

(h) The matrix of a quadratic form is

(i) symmetric

(ii) anti-symmetric

(iii) orthogonal

(iv) Hermitian

2. Answer the following : 4×4=16

(a) Solve the equation

$$x^4 - 7x^3 + 27x^2 - 47x + 26 = 0$$

given that one of its root is $2 + 3i$.

(b) If $\begin{bmatrix} A^{-1} & 0 \\ X & A^{-1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ B & A \end{bmatrix}^{-1}$ and A is non-singular, then find out the value of X.

(c) Show that

$$\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ca \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

(d) Reduce the matrix A to its normal form, where

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

Hence find the rank of A.

3. Answer any *two* of the following : 7×2=14

(a) (i) Write the statements of remainder theorem and factor theorem of classical algebra. 4

(ii) If the sum of two roots of the cubic

$$x^3 + a_1x^2 + a_2x + a_3 = 0$$

is zero, prove that $a_1a_2 = a_3$. 3

(b) *(i)* If $(ax^3 + bx^2 + cx + d)$ be divisible by $(x^2 + l^2)$, then show that $ad = bc$. 3½

(ii) Find the equation whose roots are 1, −2, 3, −4. 3½

(c) *(i)* Solve the equation

$$x^3 - 9x^2 + 23x - 15 = 0$$

whose roots are in AP. 3½

(ii) If the equation

$$x^4 + ax^3 + bx^2 + cx + d = 0$$

has three equal roots, then show that each of them is equal to

$$\frac{6c - ab}{3a^2 - 8b}$$

3½

(d) *(i)* Prove that, if two vectors are linearly dependent, one of them is a scalar multiple of the other. 4

(ii) Define basis of a vector space. 3

4. Answer any *two* of the following : 4½×2=9

(a) If A and B are two idempotent matrices, then show that $A + B$ will be idempotent if $AB = BA = 0$.

(b) Show that any diagonal element of a Hermitian matrix is necessarily real.

(c) If A and B are two n-rowed orthogonal matrices, then AB and BA are also orthogonal matrices.

(d) Prove that the necessary and sufficient condition for a square matrix A to possess the inverse is that $|A| \neq 0$.

(e) Let A and B be two square matrices of order n. Then prove that

$$\text{tr}(AB) = \text{tr}(BA)$$

5. Answer any *four* of the following : 6×4=24

(a) Write the general definition of determinant and mention its properties.

(b) Define adjoint of a square matrix. If A is a square matrix of order n, then prove that

$$A(\text{adj } A) = (\text{adj } A)A = |A| I_n$$

(c) Solve the following equations by using Cramer's rule :

$$x+2y+3z=6$$
$$2x+4y+z=7$$
$$2x+2y+9z=14$$

(d) Define circulant determinant and Vandermonde determinant for *n*th order.

(e) Investigate for what values of λ and μ, the system of simultaneous equations

$$x+y+z=6$$
$$x-2y+3z=10$$
$$x+2y+\lambda z=\mu$$

has *(i)* no solution, *(ii)* a unique solution and *(iii)* an infinite number of solutions.

(f) Given that

$$A=\begin{bmatrix}1 & -2 & -1\\ 2 & 3 & 1\\ 0 & 5 & -2\end{bmatrix}$$

Compute *(i)* $|A|$, *(ii)* adj A and *(iii)* A^{-1}.

6. Answer any *two* of the following : 4½×2=9

(a) Prove that rank of a non-singular matrix is equal to the rank of its reciprocal matrix.

(b) Under what condition the rank of the following matrix A is 3? Is it possible for the rank to be 1? Why?

$$A=\begin{bmatrix}2 & 4 & 2\\ 3 & 1 & 2\\ 1 & 0 & x\end{bmatrix}$$

(c) Obtain a *g*-inverse of the matrix given by

$$A=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix}$$

(d) Find the characteristic roots and characteristic vectors of the matrix

$$A=\begin{bmatrix}a & h & g\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$$

(e) State and prove Cayley-Hamilton theorem.

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