Total No. of Printed Pages—11

2 SEM TDC STSH (CBCS) C 3 (N/O)

2023

(May/June)

STATISTICS

(Core)

Paper : C–3

(Probability and Probability Distributions)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the following alternatives : 1×5=5

(a) If A_1, A_2 and A_3 are three mutually exclusive events, then the probability of their union is equal to

(i) $P(A_1)P(A_2)P(A_3)$

(ii) $P(A_1)+P(A_2)+P(A_3)-P(A_1 A_2 A_3)$

(iii) $P(A_1)+P(A_2)+P(A_3)$

(iv) $P(A_1)P(A_2)+P(A_1)P(A_3)+P(A_2)P(A_3)$

(b) If X is a random variable and a, b are constants, then $V(aX+b)$ is

(i) $abV(X)$

(ii) $aV(X)$

(iii) $a^2V(X)$

(iv) $bV(X)$

(c) If X is a random variable, then $E[e^{itX}]$ is known as

(i) characteristic function

(ii) moment generating function

(iii) probability generating function

(iv) None of the above

(d) The number of parameters of the normal distribution is

(i) 1

(ii) 2

(iii) 3

(iv) 4

(e) Binomial distribution tends to Poisson distribution when

(i) $n \to 0$, $p \to 0$, $np \to 0$

(ii) $n \to \infty$, $p \to \infty$, $np \to 0$

(iii) $n \to \infty$, $p \to \infty$, $np \to \mu$

(iv) $n \to \infty$, $p \to 0$, $np \to \mu$

2. Answer the following questions : 2×5=10

(a) Write down the classical definition of probability.

(b) Two fair dice are thrown simultaneously. What is the probability that the sum of numbers on the dice exceeds 8?

(c) Define marginal density function of a random variable X.

(d) Obtain the moment generating function of Poisson distribution.

(e) Prove that if X and Y are independent random variables, then

$$E(XY) = E(X) \cdot E(Y)$$

3. (a) State Bayes' theorem. In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total. Of their output, 5%, 4% and 2% are defective bolts respectively. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C? 2+4=6

Or

(b) (i) State the addition theorem of probability. 3

(ii) The probabilities of solving a problem by A and B are respectively $\frac{1}{10}$ and $\frac{1}{12}$. What is the probability that the problem will be solved if it is assigned to them? 3

4. (a) Define discrete random variable. A random variable X has the following probability distribution :

Value of X	0	1	2	3	4	5	6	7
P(X)	0	k	$2k$	k	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Find k.

(ii) Find $P(X<6)$. 3+5=8

Or

(b) Two random variables X and Y have the following joint density function :

$$f(x, y) = 2 - x - y;\ 0 < x < 1,\ 0 \le y \le 1$$
$$= 0 \qquad , \text{otherwise}$$

Find (i) marginal probability distribution of X and Y, and (ii) conditional density functions of $X|Y$ and $Y|X$. 8

5. (a) Define mathematical expectation of a random variable. Show that the mathematical expectation of sum of two random variables is the sum of their individual expectations. Can it be extended? 2+4+2=8

Or

(b) Define cumulant generating function. Show that the rth cumulant for the distribution

$$f(x) = ke^{-kx};\ 0 < x < \infty,\ k \neq 0 \in R$$

is $\frac{1}{k^r}\lfloor r-1$. 3+5=8

6. (a) Define Binomial distribution with parameters n and p. Give one example of binomial distribution. From the m.g.f. or otherwise, obtain the mean of this distribution. 2+2+5=9

Or

(b) Define geometric distribution. Obtain mean and variance of this distribution. 3+6=9

7. (a) Under what conditions, does the binomial distribution tend to normal distribution? What is the m.g.f. of normal distribution? State some important properties of normal distribution. 2+2+5=9

Or

(b) Define beta distribution of first kind and derive its mean. 3+6=9

(Old Course)

Full Marks : 50
Pass Marks : 20

Time : 2 hours

1. Choose the correct answer from the following alternatives : 1×5=5

(a) If A_1, A_2 and A_3 are three mutually exclusive events, then the probability of their union is equal to

(i) $P(A_1)P(A_2)P(A_3)$

(ii) $P(A_1)+P(A_2)+P(A_3)-P(A_1 A_2 A_3)$

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(i) characteristic function

(ii) moment generating function

(iii) probability generating function

(iv) None of the above

(d) The number of parameters of the normal distribution is

(i) 1

(ii) 2

(iii) 3

(iv) 4

(e) Binomial distribution tends to Poisson distribution when

(i) $n \to 0,\ p \to 0,\ np \to 0$

(ii) $n \to \infty,\ p \to \infty,\ np \to 0$

(iii) $n \to \infty,\ p \to \infty,\ np \to \mu$

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2. Answer the following questions : 2×5=10

(a) Write down the classical definition of probability.

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3. (a) State Bayes' theorem. In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total. Of their output, 5%, 4% and 2% are defective bolts respectively. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C? 1+4=5

Or

(b) (i) State the addition theorem of probability. 2

(ii) The probabilities of solving a problem by A and B are respectively $\frac{1}{10}$ and $\frac{1}{12}$. What is the probability that the problem will be solved if it is assigned to them? 3

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(i) Find k.

(ii) Find $P(X < 6)$. 2+5=7

Or

(b) Two random variables X and Y have the following joint density function :

$$f(x, y) = 2 - x - y; \quad 0 < x < 1, \; 0 \le y \le 1$$
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5. (a) Define mathematical expectation of a random variable. Show that the mathematical expectation of sum of two random variables is the sum of their individual expectations. Can it be extended? 2+4+1=7

Or

(b) Define cumulant generating function. Show that the rth cumulant for the distribution

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