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2 SEM TDC MTMH (CBCS) C 4

2023

(May/June)

MATHEMATICS

(Core)

Paper : C–4

(Differential Equations)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Write one example where mathematical model can be used. 1

(b) Write the order of the differential equation

$$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3+y=x^4$$ 1

(c) Classify the differential equation

$$\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 2y^3 = x$$

as linear or non-linear. 1

(d) If for the equation $\frac{dy}{dx} = x$, $y(1) = 2$, then find the value of $y(2)$. 2

(e) Justify that for real values of x, function defined by $y = f(x) = 2\sin x + 3\cos x$ is an explicit solution of the differential equation

$$\frac{d^2y}{dx^2} + y = 0$$ 4

Or

Show that $\frac{dy}{dx} = 3x^2$ has an infinite family of functions as solutions.

3×2=6

(f) Solve (any *two*) :

(i) $\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$

(ii) $(3x + 2y)dx + (2x + y)dy = 0$

(iii) $x\frac{dy}{dx} - 2y = 2x^4$

(iv) $(x + y)\,dx - x\,dy = 0$

(Continued)

2. (a) State balance law of modelling. 1

(b) Write one example of applying compartmental notion in modelling. 1

(c) Write the word equation of modelling in births and deaths in a population. 2

(d) Draw the input-output compartmental diagram for drug assimilation model. 2

(e) Derive the differential equation for radioactive decay. 4

Or

Describe lake pollution model.

3. (a) Write when

$$a(x)\frac{d^2y}{dx^2} + b(x)\frac{dy}{dx} + c(x)y = f(x)$$

will become a homogeneous equation. 1

(b) Write when the solution of an nth order homogeneous linear differential equation will have a trivial solution. 1

(c) Write the number of arbitrary constants appearing in the solution of a third-order ordinary differential equation. 1

(Turn Over)

(d) Show that e^{-x}, e^{3x}, e^{4x} are linearly independent solutions of

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 12y = 0$$ 4

Or

Solve :

$$\frac{d^2y}{dx^2} + 9y = 0$$

(e) Solve (any *one*) : 5

(i) $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 4e^x$

(ii) $\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 7\frac{dy}{dx} - 2y = e^{2x}\cosh x$

4. (a) Justify that the solution of

$$\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + x^2y = e^{2x}$$

exists and unique for all $x \in R$. 2

(b) Solve (any *one*) : 5

(i) $\frac{d^2y}{dx^2} - y = x^2\cos x$

(ii) $\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = x^2\cos^2 x$

(c) Solve by using the method of variation of parameters (any *one*) : 6

(i) $\frac{d^2y}{dx^2} + y = x$

(ii) $\frac{d^2y}{dx^2} + n^2y = \sec nx$

5. (a) Define equilibrium point in phase plane. 2

(b) Answer any *two* from the following questions : 4×2=8

(i) Write about interpretation of the phase plane.

(ii) Formulate the differential equation to study outbreak of cholera.

(iii) Write the assumptions considered in predator-prey model.

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