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2 SEM TDC MTMH (CBCS) C 3

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(May/June)

MATHEMATICS

(Core)

Paper : C-3

(**Real Analysis**)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. (a) If $a \neq 0$, $b \neq 0$, then show that

$$\frac{1}{(ab)} = \left(\frac{1}{a}\right)\left(\frac{1}{b}\right), \quad a, b \in \mathbb{R} \quad 3$$

- (b) Prove that, if x is a rational number and y is an irrational number, then $x+y$ is an irrational number. 3

(2)

(c) If $f(x) = \frac{2x^2 + 3x + 1}{2x - 1}$ for $2 \leq x \leq 3$, find a constant M such that $|f(x)| \leq M$ for all x satisfying $2 \leq x \leq 3$. 3

(d) State the supremum property of real numbers \mathbb{R} . 1

(e) If $S = \left\{ 1 - \frac{(-1)^n}{n}; n \in \mathbb{N} \right\}$, then find $\inf S$ and $\sup S$. 4

Or

Let S be a non-empty bounded set in \mathbb{R} . Let $a < 0$ and $aS = \{as : s \in S\}$. Prove that

$$\inf(aS) = a \sup S, \quad \sup(aS) = a \inf S$$

(f) Prove that an upper bound u of a non-empty set S in \mathbb{R} is the supremum of S if and only if for every $\varepsilon > 0$ there exists an $s_\varepsilon \in S$ such that $u - \varepsilon < s_\varepsilon$. 3

(g) If x and y are any real numbers with $x < y$, then prove that there exists a rational number $r \in \mathbb{Q}$ such that $x < r < y$. 3

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(3)

(h) Prove that the set \mathbb{R} of real numbers is not countable. 5

Or

If $I_n = [a_n, b_n]$, $n \in \mathbb{N}$ is a nested sequence of closed, bounded intervals such that the lengths $b_n - a_n$ of I_n satisfy $\inf\{b_n - a_n : n \in \mathbb{N}\} = 0$, then prove that the number ξ contained in I_n , $\forall n \in \mathbb{N}$ is unique.

(i) Show that if $a, b \in \mathbb{R}$ and $a \neq b$, then there exists ε -neighbourhoods U of a and V of b such that $U \cap V = \emptyset$. 5

Or

Prove that there does not exist a rational number r such that $r^2 = 2$.

2. (a) Define range of a real sequence. 1

(b) Write the limit point of the sequence (S_n) , where

$$S_n = (-1)^n \left(1 + \frac{1}{n} \right), \quad n \in \mathbb{N} \quad 2$$

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(4)

(c) Every convergent sequence is bounded.
Is the converse true? Justify. 1+2=3

(d) Prove that every bounded sequence has
a limit point. 4

Or

Prove that $\lim_{n \rightarrow \infty} \left(\frac{1}{1+na} \right) = 0, a > 0.$

(e) Let the sequence $X = (x_n)$ converge to x .
Prove that the sequence $(|x_n|)$ of
absolute values converges to $|x|$. 4

(f) Define subsequence of a sequence of
real numbers. 2

(g) If a sequence $X = (x_n)$ of real numbers
converges to x , then prove that any
subsequence $X' = (x_{n_k})$ of X also
converges to x . 4

(h) Show that the sequence (e_n) , where

$$e_n = \left(1 + \frac{1}{n} \right)^n, n \in \mathbb{N}$$

is convergent. 5

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(Continued)

(5)

Or

State and prove Bolzano-Weierstrass
theorem.

(i) Show that the sequence (x_n) , where

$$x_n = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!}, n \in \mathbb{N}$$

is convergent. 5

Or

Show that the sequence (x_n) , where

$$x_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

cannot converge.

3. (a) State the necessary condition for
convergence of an infinite series. 1

(b) State True or False : 1

In convergent series, brackets may be
inserted at will without affecting
convergence but may not be removed.

(c) Discuss the convergence of a geometric
series. 4

4

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(6)

Or

Investigate the behaviour of the series whose n th term is $\sin\left(\frac{1}{n}\right)$.

- (d) Prove that the p -series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges when $p > 1$ and diverges when $0 < p \leq 1$. 4

Or

Establish the convergence or divergence of the infinite series whose n th term is

$$\{(n^3 + 1)^{1/3} - n\}$$

- (e) Define alternating series and conditionally convergent series. 2
- (f) State the conditions of Leibnitz test. 2
- (g) Test the convergence of the following (any two) : $3 \times 2 = 6$

(i) $\frac{1 \cdot 2}{3^2 \cdot 4^2} + \frac{3 \cdot 4}{5^2 \cdot 6^2} + \frac{5 \cdot 6}{7^2 \cdot 8^2} + \dots$

(7)

(ii) $\frac{1}{1+2} + \frac{2}{1+2^2} + \frac{3}{1+2^3} + \dots$

(iii) $1 + \frac{4}{2!} + \frac{4^2}{3!} + \frac{4^3}{4!} + \dots$

(iv) $\frac{1}{x} + \frac{1}{x-1} + \frac{1}{x+1} + \frac{1}{x-2} + \frac{1}{x+2} \dots$,
 x being a positive fraction.
