

6 SEM TDC DSE MTH (CBCS) 6 (H)

2023

(May/June)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE-6

(**Mathematical Methods**)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) If T is the period of the function $f(x)$, then write the period of $f(ax)$, where $a \neq 0$. 1
- (b) Write two Dirichlet conditions for Fourier expansion of a function. 2
- (c) Find a Fourier series for the function $f(x) = x^2$ in the interval $(0, 2\pi)$. 7

(Turn Over)

Or

Find the Fourier series for the function
 $f(x) = x \sin x$ in $(-\pi, \pi)$.

2. (a) Write the value of (i) $L\{t\}$ and
 (ii) $L\{\cos 3t\}$. 1+1=2
- (b) Find $L\{\cos^2 t\}$. 2
- (c) If $f(s) = L\{F(t)\}$, then prove that
 $L\{e^{at}F(t)\} = f(s-a)$. 4

Or

Find $L\{te^{at} \sin at\}$.

- (d) Find any three of the following : 4×3=12
- (i) $L\{\sinh^2 t\}$
- (ii) $L\{(t+2)^3\}$
- (iii) $L\{e^{3t} \sin 2t\}$
- (iv) $L\{t \cos^2 t\}$
- (v) $L\{t^2 \cos at\}$

3. (a) Write the value of (i) $L^{-1}\left\{\frac{S}{S^2+4}\right\}$ and
 (ii) $L^{-1}\left\{\frac{1}{S^4}\right\}$. 1+1=2
- (b) Find $L^{-1}\left\{\frac{S+2}{(S+2)^2+6^2}\right\}$. 2

- (c) Find any two of the following : 3×2=6

(i) $L^{-1}\left\{\frac{S}{(S+3)^{7/2}}\right\}$

(ii) $L^{-1}\left\{\frac{e^{-\pi S}}{S^2+4}\right\}$

(iii) $L^{-1}\left\{\frac{e^{-3S}}{(S-4)^2}\right\}$

(iv) $L^{-1}\left\{\log \frac{S+4}{S+2}\right\}$

4. (a) Write the Fourier cosine integral formula. 1
- (b) Define Fourier transform of a function. 1
- (c) State and prove the linear property of Fourier transform. 5

Or

Find the Fourier sine transform of

$$2e^{-5x} + 5e^{-2x}$$

- (d) Find the Fourier cosine transform of e^{-x^2} . 8

Or

Find the Fourier transform of

$$f(x) = xe^{-x}, 0 \leq x < \infty$$

- (e) Find the Fourier transform of

$$f(x) = \begin{cases} \frac{1}{2x}, & \text{for } |x| \leq a \\ 0, & \text{for } |x| > a \end{cases}$$

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Or

- Find the Fourier transform of

$$f(x) = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & \text{otherwise} \end{cases}$$

- (f) Write the formula for inverse Fourier transform of a function $f(x)$. 2
5. (a) Write the value of $L\left\{\frac{\partial^2 y}{\partial t^2}\right\}$. 1
- (b) Find the value of $L\left\{\frac{\partial^2 y}{\partial x^2}\right\}$. 2
- (c) Solve any two of the following : $6 \times 2 = 12$
- (i) $\frac{d^2 y}{dt^2} + \frac{dy}{dt} = t^2 + 2t$, $y(0) = 4$, $y'(0) = 2$
- (ii) $\frac{d^2 y}{dt^2} + 25y = 10 \cos 5t$,
 $y(0) = 2$, $y'(0) = 0$
- (iii) $t \frac{d^2 y}{dt^2} + \frac{dy}{dt} + 4ty = 0$, $y(0) = 3$, $y'(0) = 0$
- (iv) $\frac{\partial y}{\partial x} - \frac{\partial y}{\partial t} = 1 - e^{-t}$, $0 < x < 1$, $t > 0$
and $y(x, 0) = x$
