6 SEM TDC DSE MTH (CBCS) 2 (H)

2023

(May/June)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–2

(Linear Programming)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1×8=8

1. Answer the following questions :

   (a) Define degenerate basic feasible solution.

   (b) Write about decision variable.

   (c) Define slack variable.

   (d) Write the standard form of primal in duality.

(Turn Over)

(e) Define symmetric primal dual problem.

(f) State the rim condition of transportation problem.

(g) Define saddle point in a game theory.

(h) What is fair game in a game theory?

2. Answer any *two* from the following : 2×2=4

(a) Write the mathematical formulation of transportation problem.

(b) Explain briefly the basic solution of linear programming problem.

(c) Describe general rule of dominance property of game theory.

3. Answer the following questions : 4×5=20

(a) Write the rule of construction of dual from primal.

(b) Write the characteristic of standard form of general linear programming problem.

(Continued)

(c) Find the dual :

$$\text{Max } Z = 4x_1 - 3x_2 + 2x_3$$

subject to

$$x_1 - 7x_2 + 3x_3 \le 6$$
$$-5x_2 + 3x_3 \ge 8$$
$$2x_1 - 4x_2 + 5x_3 = 7$$

$x_1, x_3 \ge 0$, x_2 is unrestricted in sign

(d) In an assignment problem, if we add (or subtract) a constant to every element of a row (or column) of the cost matrix $[c_{ij}]$, then show that an assignment plan that minimizes the total cost for new cost matrix also minimizes the total cost for the original cost matrix.

(e) Find the range of the values of p and q which will render the entry (2, 2), a saddle point for the game :

Player B

Player A	B_1	B_2	B_3
A_1	2	4	5
A_2	10	7	q
A_3	4	p	6

4. (a) Prove that dual of the dual is primal itself. 5

(Turn Over)

Or

If x^* and w^* be any two feasible solutions of the primal, Max $Z_x = cx$, subject to $Ax \leq b$, $x \geq 0$ and corresponding dual, Min $Z_w = b'w$, subject to $A'w \geq c'$, $w \geq 0$ respectively and $cx^* = b'w^*$, then x^* and w^* are optimal feasible solutions of the primal and dual respectively. Prove it.

(b) Solve the pay-off matrix with respect to player A by using dominance property : 5

		Player B				
		1	2	3	4	5
Player A	1	4	6	5	10	6
	2	7	8	5	9	10
	3	8	9	11	10	9
	4	6	4	10	6	4

5. Answer any *one* of the following : 6

(a) Find the optimal assignment of the corresponding assignment cost from the following cost matrix :

	A	B	C	D	E
I	9	8	7	6	4
II	5	7	5	6	8
III	8	7	6	3	5
IV	8	5	4	9	3
V	6	7	6	8	5

(Continued)

(b) Find the optimal assignment profit from the following profit matrix :

	D_1	D_2	D_3	D_4	D_5
O_1	2	4	3	5	4
O_2	7	4	6	8	4
O_3	2	9	8	10	4
O_4	8	6	12	7	4
O_5	2	8	5	8	8

6. Answer any *two* of the following : 8×2=16

(a) Solve by Big-M method :

$$\text{Max } Z = -2x_1 - x_2$$

subject to

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

(b) Solve :

$$\text{Min } Z = x_1 - 3x_2 + 2x_3$$

subject to

$$3x_1 - x_2 + 2x_3 \leq 7$$
$$-2x_1 + 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

(Turn Over)

(c) Solve by two-phase method :

$$\text{Min } Z = \frac{15}{2}x_1 - 3x_2$$

subject to

$$3x_1 - x_2 - x_3 \geq 3$$
$$x_1 - x_2 + x_3 \geq 2$$
$$x_1, x_2, x_3 \geq 0$$

7. Answer any *one* of the following : 9

(a) Determine the initial basic feasible solution to the following transportation problem by least cost method and then find the optimal solution :

	D_1	D_2	D_3	D_4	a_i
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
b_j	16	18	31	25	

where O_i and D_j denote the ith origin and jth destination respectively.

(b) Find the initial basic feasible solution using VAM and find the optimal solution :

	A	B	C	D	a_i
S_1	8	9	6	3	18
S_2	6	11	5	10	20
S_3	3	8	7	9	18
b_j	15	16	12	13	

(Continued)

8. Answer any *one* of the following : 7

(a) Obtain the optimal strategies of each player from the pay-off matrix :

		Player B			
		I	*II*	*III*	*IV*
Player A	*I*	3	2	4	0
	II	3	4	2	4
	III	4	2	4	6
	IV	0	4	0	8

(b) Player A can choose his strategies from A_1, A_2 and A_3 only while player B can choose from B_1, B_2 only. The rule of game states that the payment should be made in accordance with the selection of strategies :

Strategy pair selected	*Payment to be made*
$A_1 B_1$	*A* to *B* ₹ 1
$A_1 B_2$	*B* to *A* ₹ 6
$A_2 B_1$	*B* to *A* ₹ 3
$A_2 B_2$	*B* to *A* ₹ 4
$A_3 B_1$	*A* to *B* ₹ 2
$A_3 B_2$	*A* to *B* ₹ 6

Find the pay-off matrix and optimal strategies of each player.

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