3 SEM TDC STS M 2 (N & O)

2021

(March)

STATISTICS

(Major)

Course : 302

(Numerical Methods)

(New and Old Course)

Full Marks : 80
Pass Marks : 24/32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct alternative out of the given ones : 1×8=8

(a) If h be the interval of differencing, then $\Delta \log x$ is

(i) $\log(x+h)$

(ii) $\log(x+h)-\log x$

(iii) $E\log(x+h)$

(iv) $\log(x+h)+\log x$

(b) If $f(x) = e^x$, then $\Delta^2 f(x)$ is equal to

(i) $(e^h - 1)e^x$

(ii) $3(e^h - 1)^2$

(iii) $3(e^h - 1)^2 e^x$

(iv) None of the above

(c) Lagrange's formula is useful for

(i) interpolation

(ii) extrapolation

(iii) inverse interpolation

(iv) All of the above

(d) Newton's divided difference formula for interpolation is used for

(i) equal interval

(ii) unequal interval

(iii) central difference

(iv) None of the above

(Continued)

(e) Inverse interpolation method can be used

(i) only when arguments are at equal interval

(ii) for both equal and unequal intervals of arguments

(iii) only when entries are at equal interval

(iv) only when both arguments and entries are at unequal interval

(f) Weddle's rule can be applied when the number of subintervals is

(i) multiple of 4

(ii) multiple of 6

(iii) multiple of 8

(iv) any positive integer

(g) Simpson's $\frac{3}{8}$th rule is derived under the assumption that $Y = f(x)$ is a polynomial of degree

(i) 2

(ii) 3

(iii) 6

(iv) 8

(h) Which one of the following is not a transcendental equation?

(i) $e^{-x} = \sin x$

(ii) $e^{x} - 3x = 0$

(iii) $2x - \log_{10} x = 7$

(iv) $2x - \log_{10} 2 = 7$

2. Answer the following in brief : 2×5=10

(a) Prove that the second difference of a polynomial of second degree is constant.

(b) What are the assumptions on which interpolation and extrapolation are based?

(c) What is inverse interpolation?

(d) What are the basic conditions to apply Simpson's $\frac{1}{3}$rd rule?

(e) Write two properties of algebraic and transcendental equations.

3. (a) Prove that the operators E and Δ are commutative w.r.t. constant. Find the polynomial function $f(x)$ whose first difference is $9x^2 + 11x + 5$. Use the method of finite differences to find

$$\sum_{x=1}^{n} u_x$$

where $u_x = x(x+2)(x+4)$. 3+4+4=11

(Continued)

Or

(b) Define the terms—arguments, entry and interval of differencing in a difference table. What is the difference between

$$\left(\frac{\Delta u_x}{E u_x}\right)^2 \text{ and } \left(\frac{\Delta^2 u_x}{E^2 u_x}\right)?$$

Prove that

$$u_0 + \frac{u_1 x}{1!} + \frac{u_2 x^2}{2!} + \frac{u_3 x^3}{3!} + \cdots$$
$$= e^x \left[u_0 + \Delta u_0 + \frac{x^2}{2!}\Delta^2 u_0 + \frac{x^3}{3!}\Delta^3 u_0 + \cdots \right]$$

3+3+5=11

4. Answer any *four* of the following : 7×4=28

(a) What are the underlying assumptions for the validity of the various methods used for interpolation? Derive Newton's backward interpolation formula. 3+4=7

(b) What are the practical advantages arising from the use of central differences in interpolation? Derive the Stirling's interpolation formula. 3+4=7

(c) What are the advantages of divided differences over ordinary differences? Prove that divided differences are symmetric functions of their arguments.

2+5=7

(d) In which situation, Lagrange's interpolation formula is used in general? By means of Lagrange's interpolation formula, prove that

$$y_1 = y_3 - 0\cdot3(y_5 - y_{-3}) + 0\cdot2(y_3 - y_{-5})$$

2+5=7

(e) Name three methods of inverse interpolation. Describe the method of successive approximation for solving the problem of inverse interpolation and give two applications of this method.

1+4+2=7

5. (a) What is meant by numerical integration? Derive trapezoidal formula from general quadrature formula. Why is it called trapezoidal?

2+3+2=7

Or

(b) Define numerical quadrature. When would you recommend it? Write down the general quadrature formula and derive any one quadrature formula which you consider as the best.

1+1+2+3=7

(Continued

6. Answer any *two* of the following : 8×2=16

(a) When do we use numerical differentiation? How do the errors occur in numerical computation of derivatives? Given the following pairs of values of x and $y = f(x)$

x	:	1	2	4	8	10
$y = f(x)$	:	0	1	5	21	27

determine numerically the first derivative of $f(x)$ at $x = 4$. 1+2+5=8

(b) Describe the regula-falsi method for solution of transcendental equations. Use this method to find the real root of the equation $x^6 - x^4 - x^3 - 1 = 0$ which lies between 1 and 2. 4+4=8

(c) In what situation the bisection method is applicable? Describe the method and mention clearly how the percentage error is used to determine the number of iterations. 2+6=8

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