1 SEM TDC STSH (CBCS) C 2

2021

(Held in January/February, 2022)

STATISTICS

(Core)

Paper : C–2

(Calculus)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following alternatives in each question : 1×8=8

(a) If f and g be two functions which are continuous at $x = a$, then the function, which will be continuous at $x = a$, is

(i) $f \pm g$

(ii) $f \cdot g$

(iii) $k \cdot f$ (k is a constant)

(iv) All of the above

(b) If $\lim_{x \to a} \phi(x) = 0$ and $\lim_{x \to a} \psi(x) = \infty$, then $\lim_{x \to a} \phi(x) \cdot \psi(x)$ is said to assume indeterminate form

(i) $\frac{0}{0}$

(ii) $\frac{\infty}{\infty}$

(iii) $0 \times \infty$

(iv) None of the above

(c) If $y = e^{-2x}$, then y_3 is

(i) $8e^{2x}$

(ii) $-8e^{-2x}$

(iii) $4e^{2x}$

(iv) None of the above

(d) The value of $\Gamma(n+1)$ is

(i) $(n+1)!$

(ii) $(n-1)\,\Gamma(n-1)$

(iii) $n\,\Gamma n$

(iv) None of the above

(Continued)

(e) The value of $\int_1^2 \int_0^{3y} y \, dy \, dx$ is

(i) 3

(ii) 5

(iii) 7

(iv) 9

(f) The integrating factor of the equation

$$\frac{dy}{dx} + \frac{y}{x} = y^2$$

is

(i) $\log(1/x)$

(ii) $1/x$

(iii) $\log x$

(iv) None of the above

(g) If $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$, then its particular integral is

(i) e^{5x}

(ii) $\frac{e^{5x}}{12}$

(iii) $\frac{e^{2x}}{10}$

(iv) None of the above

(h) If $z = ax + by + ab$, then the partial differential equation is

(i) $z = x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x}$

(ii) $z = x\frac{\partial z}{\partial y} + \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right) + y\frac{\partial z}{\partial x}$

(iii) $z = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)$

(iv) None of the above

2. Answer the following questions :

(a) Examine the continuity of the following function at $x = 2$: 3

$$f(x) = \begin{cases} (x^2 / a) - a & , \text{ for } 0 < x < a \\ 0 & , \text{ for } x = 0 \\ a - (a^3 / x^3) & , \text{ for } x > 0 \end{cases}$$

(b) If

$$u_1 = \frac{x_2 x_3}{x_1}, u_2 = \frac{x_3 x_1}{x_2}, u_3 = \frac{x_1 x_2}{x_3}$$

then show that, $J(u_1, u_2, u_3) = 4$. 3

(c) Evaluate : 4

$$\int_0^{\pi/2} \log \sin x \, dx$$

(d) Solve : 4

$$(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$$

(e) Find the partial differential equation by eliminating a and b from

$$z = ax + by + a^2 + b^2$$ 2

3. Answer any *three* questions : 7×3=21

(a) Define right-hand limit and left-hand limit of a function. Find

$$\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$$

(b) What do you mean by partial differentiation? If

$$u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}, \ x^2 + y^2 + z^2 \neq 0$$

then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

(c) Give the statement of L-Hospital's rule. Evaluate

$$\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$$

(d) Find the nth derivative of $y = e^{2x} x^3$.

(e) What is a homogeneious function? State and prove Euler's theorem on homogeneous function.

(f) If $u = \frac{x^3 - y^3}{x^2 + y^2}$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$$

(g) Find for what value(s) of x the function $f(x) = 41 - 72x - 18x^2$ attains its maximum.

4. Answer any *two* questions : 7×2=14

(a) Evaluate :

$$\int_0^2 \int_0^{\sqrt{4+x^2}} \frac{dx\,dy}{4+x^2+y^2}$$

(b) Using the properties of definite integrals, show that

$$\int_0^{\pi/2} \frac{\sin x - \cos x}{\sin x + \cos x} dx = 0$$

(c) Write different forms of beta function. Show that beta function is symmetric.

(d) Show that

$$\int_0^{\pi/2} \sin^p \theta \cos^q \theta \, d\theta = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$$

where, $p > -1$, $q > -1$ and hence deduce that

$$\int_0^2 x^4 (8 - x^3)^{-1/3} dx = \frac{16}{3}\beta\left(\frac{5}{3}, \frac{2}{3}\right)$$

(e) Prove that

$$\Gamma(1+n)\Gamma(1-n) = \frac{n\pi}{\sin n\pi}$$

and hence find the value of $\Gamma\frac{1}{2}$.

5. Answer any *two* questions : 7×2=14

(a) Write the general form of a first-order and first-degree differential equation. Discuss the working rule for solving a differential equation of the form

$$f_1(x)dx = f_2(y)dy$$

(b) What is the working rule for solving an exact diferential equation? Solve

$$(4x+3y+1)dx+(3x+2y+1)dy = 0$$

(c) What is an integrating factor? Solve the following differential equation :

$$(xy^2+2x^2y^3)dx+(x^2y-x^3y^2)dy = 0$$

(d) What is a linear differential equation? Solve

$$\frac{d^3y}{dx^3}+6\frac{d^2y}{dx^2}+3\frac{dy}{dx}-10y = 0$$

(e) Write the integrability condition of total differential equation. Solve

$$(y+z)dx+dy+dz = 0$$

(Turn Over)

6. Answer any *one* question : 7

(a) Distinguish between partial and ordinary differential equations. Solve the following differential equation by direct integration method :

$$X\frac{\partial^2 Z}{\partial X^2} = \frac{\partial Z}{\partial X}$$

(b) Discuss the Lagrange's method to solve partial differential equation of the form

$$Pp + Qq = R$$

Solve the following differential equation by Lagrange's method :

$$(Y + Z)p - (X + Z)q = X - Y$$

(c) What do you mean by linear homogeneous partial differential equation with constant coefficient? Solve the following :

$$\frac{\partial^2 z}{\partial x^2} = a^2\left(\frac{\partial^2 z}{\partial y^2}\right)$$

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