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**6 SEM TDC DSE MTH (CBCS) 6 (H)**

**2022**

( June/July )

**MATHEMATICS**

( Discipline Specific Elective )

( For Honours )

Paper : DSE-6

( **Mathematical Methods** )

Full Marks : 80  
Pass Marks : 32

Time : 3 hours

*The figures in the margin indicate full marks for the questions*

1. (a) Write the period of  $\sin 2x$ . 1
- (b) Write two sufficient conditions for a function to be expanded in a Fourier series. 2
- (c) Find the Fourier series for the function  $f(x) = \pi + x$  in the interval  $(0, 2\pi)$ . 7

Or

Find the Fourier series for the function  $f(x) = x$  in the interval  $(0, \pi)$ .

2. (a) Write the value of  $L\{t^2\}$ . 1
- (b) Write the value of  $L\{e^{4t}\}$ . 1
- (c) Find  $L\{\sin 2t\}$ . 2
- (d) Find any four of the following :  $4 \times 4 = 16$
- (i)  $L\{\cosh^2 2t\}$
  - (ii)  $L\{e^{3t} \cos 2t\}$
  - (iii)  $L\{(t^2 + 1)^2\}$
  - (iv)  $L\{(1 + te^t)^3\}$
  - (v)  $L\{\sin^3 2t\}$
  - (vi)  $L\{t \sin^2 2t\}$
3. (a) Write the value of  $L^{-1}\{2\}$ . 1
- (b) Write the value of  $L^{-1}\left\{\frac{S}{S^2 + 9}\right\}$ . 1
- (c) Find any two of the following :  $4 \times 2 = 8$
- (i)  $L^{-1}\left\{\frac{1}{(S+1)(S+2)}\right\}$
  - (ii)  $L^{-1}\left\{\frac{S^2}{(S+2)^3}\right\}$

(iii)  $L^{-1} \left\{ \frac{S}{(S+1)^{\frac{3}{2}}} \right\}$

(iv)  $L^{-1} \left\{ \log \frac{S+6}{S+2} \right\}$

4. (a) Write the Fourier sine integral formula. 1

(b) Write the Dirichlet's conditions of Fourier transform. 2

(c) Answer any *three* of the following :  $5 \times 3 = 15$

(i) State and prove the change of scale property of Fourier transform.

(ii) Find the Fourier transform of

$$f(x) = e^{-\frac{x^2}{2}}, \quad -\infty < x < \infty$$

(iii) Find the inverse Fourier transform of  $e^{-|s|y}$ .

(iv) Show that

$$F\{x^n f(x)\} = (-i)^n \frac{d^n}{ds^n} (\bar{f}(s))$$

(d) Find the Fourier sine and cosine transforms of  $f(x) = x$ . 7

Or

Find the Fourier transform of

$$f(x) = \begin{cases} \cos x, & \text{for } |x| < a \\ 0, & \text{for } |x| > a > 0 \end{cases}$$

( Turn Over )

5. (a) Write the value of  $L\left\{\frac{\partial y}{\partial t}\right\}$ . 1

(b) Find  $L\left\{\frac{\partial y}{\partial x}\right\}$ . 2

(c) Solve any two of the following, using Laplace transform :  $6 \times 2 = 12$

(i)  $\frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} + 5y = 5, y(0) = y'(0) = 0$

(ii)  $\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} - 3y = \sin t,$

$y(0) = y'(0) = 0$

(iii)  $\frac{d^2 y}{dt^2} + y = 0, y(0) = 1, y'(0) = 0$

(iv)  $\frac{dx}{dt} + y = \sin t, \frac{dy}{dt} + x = \cos t$

$x(0) = 2, y(0) = 2$

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