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3 SEM TDC MTH M 2

2021

(March)

MATHEMATICS

(Major)

Course : 302

(Coordinate Geometry and Algebra—I)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Coordinate Geometry)

SECTION—I

(2-Dimension)

1. *(a)* What do you mean by inverse translation? 1

(b) Find the equation of the line $3x+4y-10=0$, when the origin is transferred to the point (2, 1). 2

(c) Transform

$$12x^2+7xy-12y^2-17x-31y-7=0$$

to rectangular axes through the point (1, −1) inclined at an angle $\tan^{-1}\left(\frac{4}{3}\right)$ to the original axes. 2

2. *(a)* Write down the equation of a pair of straight lines passing through the origin. 1

(b) Find out the equation of bisectors of the angles between the pair of lines represented by $ax^2+2hxy+by^2=0$. 3

(c) For what values of k will the equation

$$3x^2+kxy-3y^2+29x-3y+18=0$$

represent a pair of straight lines? 3

(d) Show that two of the straight lines represented by the equation

$$ax^3+3bx^2y+3cxy^2+dy^3=0 \quad (a\neq 0,\ d\neq 0)$$

are at right angles, if

$$a^2+3ac+3bd+d^2=0$$ 5

Or

If

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of straight lines equidistant from the origin, then prove that

$$f^4 - g^4 = c(bf^2 - ag^2)$$

3. (a) Write down the condition satisfied by a central conic. 1

(b) Prove that the equation of the tangent to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy = 0$$

at the origin is $gx + fy = 0$. 3

(c) Reduce the equation

$$5x^2 - 24xy - 5y^2 + 4x + 58y - 59 = 0$$

to the standard form. 6

Or

Define conjugate diameters of a conic. Find the condition that the pair of lines

$$Ax^2 + 2Hxy + By^2 = 0$$

may have conjugate diameters of the conic $ax^2 + 2hxy + by^2 = 1$.

SECTION—II

(3-Dimension)

4. *(a)* On which plane, does the line $x = 0$ lie? 1

(b) A line makes angles 60° and 45° with the y-axis and z-axis. Find the direction cosines of the line. 2

(c) Prove that the line joining the points (2, 3, –2) and (3, 1, 1) is parallel to the line joining the points (2, 1, –5) and (4, –3, 1). 3

Or

Find the intercepts made on the axes by the plane

$$3x - 4y + 6z - 12 = 0$$

(d) Find the equation of the plane which passes through the point (2, –3, 1) and is perpendicular to the join of the points (4, 5, –2) and (2, –1, 6). 4

Or

Find the equation of the plane which passes through the point (2, 1, 4) and is perpendicular to the planes $9x - 7y + 6z + 48 = 0$ and $x + y - z = 0$.

5. *(a)* Define the shortest distance between two lines. Find the shortest distance between the x-axis and the line

$$\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$$ 3

(b) Find the length and equations of the line of the shortest distance between the lines

$$\frac{x+3}{-4}=\frac{y-6}{3}=\frac{z}{2} \text{ and } \frac{x+2}{-4}=\frac{y}{1}=\frac{z-7}{1}$$ 5

Or

Prove that the shortest distance between the y-axis and the line

$$\frac{x-1}{5}=\frac{y-7}{-4}=\frac{z+3}{12}$$

is $\frac{17}{13}$.

GROUP—B

(Algebra—I)

6. *(a)* What do you mean by an algebraic system? 1

(b) Define a monoid with an example. 1

(c) Show that the identity element in a group is unique. 2

(d) Answer any *two* questions : 3×2=6

(i) If $(G, *)$ is a group, then show that $(a^{-1})^{-1} = a \ \forall a \in G$.

(ii) Prove that in a group G, the equations $a * x = b$, $y * a = b$, $\forall a, b \in G$ have unique solutions in G.

(iii) Show that the set $\{1, -1, i, -i\}$ is an Abelian finite group of order 4 under multiplication.

7. Answer any *two* questions : 5×2=10

(a) Let H be a non-empty subset of a group G, then show that H is a subgroup of G iff $a, b \in H \Rightarrow ab^{-1} \in H$, where b^{-1} is the inverse of b in G.

(b) If H, K are two subgroups of an Abelian group G, then show that HK is a subgroup of G.

(c) The union of two subgroups of a group G is a subgroup of G iff one is contained in the other. Prove it.

8. (a) Define cyclic permutation of group. 1

(b) Prove that the order of each subgroup of a finite group is a divisor of the group. 4

Or

State and prove Cayley's theorem.

9. Answer any *two* questions : 5×2=10

(a) Define isomorphism of groups. If $f: G \to G'$ is an isomorphism of groups, then show that the order of an element $a \in G$ is equal to the order of the f-image of a, i.e., $o(a) = o[f(a)]$.

(b) Prove that a subgroup H of a group G is a normal subgroup of G if and only if, every left coset of H in G is a right coset of H in G.

(c) Prove that every quotient group of a cyclic group is cyclic, but the converse is not true.

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