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2 SEM TDC MTMH (CBCS) C 4

2022

(June/July)

MATHEMATICS

(Core)

Paper : C–4

(Differential Equations)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Throughout the paper, notations $y'' = \frac{d^2y}{dx^2}$, $y' = \frac{dy}{dx}$)

1. (a) Define an integrating factor of a differential equation. 1

   (b) Define implicit solution of the differential equation. 1

(Turn Over)

(c) Show that the function f defined by $f(x) = 2e^{3x} - 5e^{4x}$, is a solution of the differential equation $y'' - 7y' + 12y = 0$. 3

Or

Show that the function $x^2 + y^2 = 25$ is an implicit solution of the differential equation $x + yy' = 0$ on the interval $-5 < x < 5$

(d) Solve the initial value problem

$$y' = e^{x+y}, y(1) = 1$$

2

(e) Verify the exactness of the differential equation, 2

$$(2x \sin y + y^3 e^x)dx + (x^2 \cos y + 3y^2 e^x)dy = 0$$

(f) Solve any *two* of the following : 3×2=6

(i) $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$

(ii) $xy' + (x+1)y = x^3$

(iii) $y' + 3x^2 y = x^2$, $y(0) = 2$

2. (a) Draw the input-output compartmental diagram for lake pollution model. Write the word equation to derive this model.

1+1=2

(b) Derive the formula for half-life of radioactive material.

2

(c) Derive the differential equation of exponentially growth population model. 3

(d) Answer any *one* of the following : 3

(i) Solve the differential equation $\frac{dC}{dt} = \frac{F}{V}(c_{in} - C)$ with initial condition $C(0) = c_0$.

(ii) How long ago was the radioactive carbon (^{14}C) formed and, within an error margin, the Lascaux Cave paintings painted? (the half-life of ^{14}C is $5{,}568 \pm 30$ years). Decay rate of carbon ^{14}C is 1·69 per minute per gram and initially 13·5 per minute per gram.

3. (a) Define linear combinations of n functions. 1

(b) State the principle of superposition for homogeneous differential equation. 1

(c) Fill in the blank :

If the Wronskian of two solutions of 2nd order differential equation is identically zero, then the solutions are linearly ____. 1

(d) Show that e^{2x} and e^{3x} are the two solutions of the equation $y'' - 5y' + 6y = 0$ and also verify the principle of superposition. 3

(Turn Over)

(e) If $y_1(x)$ and $y_2(x)$ are any two solutions of the equation

$$a_0(x)y'' + a_1(x)y' + a_2(x)y = 0,$$
$$a_0(x) \neq 0, \ x \in (a, \ b)$$

then prove that the linear combination $c_1y_1(x) + c_2y_2(x)$, where c_1 and c_2 are constants, is also a solution of the given equation. 4

Or

Show that $e^x \sin x$ and $e^x \cos x$ are linearly independent solutions of $y'' - 2y' + 2y = 0$. Write the general solution. Find the solution $y(x)$ with the property $y(0) = 2$, $y'(0) = -3$.

4. Answer any *one* of the following : 5

(a) If $y = x$ is a solution of $(x^2 + 1)y'' - 2xy' + 2y = 0$, then find a linearly independent solution by reducing the order.

(b) Solve $x^2y'' - 2xy' + 2y = x^3$

5. Answer any *two* of the following : 5×2=10

(a) Solve $y'' + ay = \sec ax$.

(b) Solve by method of undetermined coefficient $y'' - 2y' + y = x^2$.

(c) Solve by method of variation of parameter

$$y'' + y = \tan x$$

6. (a) Define equilibrium solution of a differential equation. 1

(b) Write the word equation and differential equation for the model of battle. 2

(c) Find the equilibrium solution of the differential equation of epidemic model of influenza. 3

(d) Draw the phase plane diagram of 4

$$dx / dt = 0 \cdot 2x - 0 \cdot 1\, xy,$$

$$dy / dt = -0 \cdot 15\, y + 0 \cdot 05\, xy$$

Or

Sketch the phase-plane trajectory and determine the direction of trajectory of model of battle.

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