1 SEM TDC MTH M 1

2021

(March)

MATHEMATICS

(Major)

Course : 101

(Classical Algebra, Trigonometry and Vector Calculus)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Classical Algebra)

1. *(a)* Write the name of the set with which the elements of a sequence can be put in a one-one correspondence. 1

(b) Write two distinct elements of $\{S_n\} = \{(-1)^n,\ n \in N\}$. 1

(c) Show that a bounded sequence may not be convergent. 3

Or

Show that the sequence $\{S_n\}$, where $S_n = \frac{1}{n}$, $n \in N$, has 0 as a limit point.

(d) Prove that every convergent sequence is bounded and has a unique limit. 5

Or

Show that the sequence $\{S_n\}$, where

$$S_n = 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n+1}$$

cannot converge.

2. (a) "If the partial sum of an infinite series is convergent, then the infinite series is divergent." State true or false. 1

(b) Define an infinite series. 2

(c) State Cauchy's root test. 2

(d) Show that the series

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots$$

is not convergent. 3

(e) Show that the sequence $\{U_n\}$, where $U_n = \sin\frac{1}{n}$, diverges. 3

Or

Show that the series

$$1+\frac{1}{2^2}+\frac{1}{3^3}+\frac{1}{4^4}+\cdots$$

converges.

(f) Test the behaviour of the series

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 2^2}+\frac{1}{3\cdot 2^3}+\frac{1}{4\cdot 2^4}+\cdots$$ 4

Or

If ΣU_n is a positive term series such that

$$\lim_{n\to\infty}\frac{U_{n+1}}{U_n}=l$$

then show that the series converges, if $l<1$.

3. (a) Write what type of equation has at least one real root. 1

(b) Write the transformed equation with sign changed of the roots of the equation

$$4x^3-11x^2+5x+3=0$$ 1

(c) Form the biquadratic equation which shall have two of its roots as i, $2i$. 2

(d) If α, β, γ be the roots of the equation

$$x^3 + px^2 + qx + r = 0$$

then find the value of $\sum\alpha^2$. 2

(e) If α and β are the roots of $x^3 - px^2 + qx - r = 0$ such that $\alpha + \beta = 0$, then show that $pq = r$. 4

Or

Solve the equation

$$x^3 - 9x^2 + 23x - 15 = 0$$

whose roots are in arithmetic progression.

(f) Find the sum of the fourth powers of the roots of the equation $x^3 - x - 1 = 0$. 5

Or

If α, β, γ are the roots of the equation

$$x^3 - px^2 + qx - r = 0$$

then form the equation whose roots are

$$\beta\gamma + \frac{1}{\alpha},\ \gamma\alpha + \frac{1}{\beta},\ \alpha\beta + \frac{1}{\gamma}$$

GROUP—B

(Trigonometry)

4. (a) Write the value of

$$\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)^2$$ 1

(b) Choose the correct answer for the following : 1

$\frac{1}{\cos\theta - i\sin\theta}$ is equal to

(i) $\cos\theta + i\sin\theta$

(ii) $\sin\theta + i\cos\theta$

(iii) $\cos\theta - i\sin\theta$

(iv) $\cos 2\theta - i\sin 2\theta$

(c) Simplify (any *one*) : 2

(i) $\frac{(\cos\theta + i\sin\theta)^5}{(\cos\theta - i\sin\theta)^3}$

(ii) $\frac{(\cos\theta - i\sin\theta)^7}{(\cos\theta + i\sin\theta)^7}$

(d) If $a = \cos 2x + i\sin 2x$, $b = \cos 2y + i\sin 2y$, then show that

$$\frac{a-b}{a+b} = i\tan(x-y)$$ 4

Or

Show that

$$(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n$$
$$=2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}$$

5. *(a)* Show that

$$e^{i\frac{\pi}{2}}=i$$ 2

(b) Resolve $e^{\sin(x+iy)}$ into real and imaginary parts. 3

Or

If $\tan(\alpha+i\beta)=x+iy$, then prove that

$$x^2+y^2+2x\cot 2\alpha=1$$

6. *(a)* Write the expansion of $\tan^{-1}x$ in terms of x, where $|x|\le 1$. 1

(b) Show that

$$\log\sec x=\frac{1}{2}\tan^2 x-\frac{1}{4}\tan^4 x+\frac{1}{6}\tan^6 x-\cdots$$ 3

Or

Find the sum of the series

$$1-\frac{1}{3\cdot 4^2}+\frac{1}{5\cdot 4^4}-\cdots$$

7. (a) Find the sum of the series (any *one*) : 4

(i) $1+\cos\alpha+c^2\cos 2\alpha+c^3\cos 3\alpha+\cdots$

(ii) $\cos\theta+\frac{\sin\theta}{1!}\cos 2\theta+\frac{\sin^2\theta}{2!}\cos 3\theta+\cdots$

(b) Show that

$$\tanh(x+y)=\frac{\tanh x+\tanh y}{1+\tanh x\tanh y}$$ 4

Or

Prove that

$$\sinh^{-1}(\cot x)=\log(\cot x+\operatorname{cosec} x)$$

GROUP—C

(Vector Calculus)

8. (a) Find $\frac{\partial}{\partial x}(x^2y\hat{i}+y^2z\hat{j}+z^2x\hat{k})$. 1

(b) A particle moves along a curve whose parametric equations are $x=2t^2+3t$, $y=3\cos t$, $z=t$. Determine its velocity and acceleration at any time. 2

(c) Let $\vec{f}(t)$ has constant magnitude. Show that $\vec{f}\cdot\frac{d\vec{f}}{dt}=0$. 2

(d) Show that

$$\nabla \cdot (\phi \vec{f}) = (\nabla \phi) \cdot \vec{f} + \phi (\nabla \cdot \vec{f})$$ 4

Or

Evaluate $\nabla(\log r)$, where

$$r = |\vec{r}|, \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

(e) Show that

$$\nabla \cdot (\nabla \phi) = \nabla^2 \phi, \phi \equiv \phi(x, y, z)$$ 4

Or

Show that curl grad $\phi = 0$.

(f) Define an irrotational vector. 2

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