Total No. of Printed Pages—7

1 SEM TDC MTH M 1

2021

(Held in January/February, 2022)

MATHEMATICS

(Major)

Course : 101

(Classical Algebra, Trigonometry and Vector Calculus)

Full Marks : 80

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Classical Algebra)

1. *(a)* Write the limit point of the sequence $\{1, 3, 1, 6, 1, 9, \cdots\}$. 1

(b) Write when a sequence is called bounded sequence. 1

(*Turn Over*)

(c) Prove that a sequence cannot converge to more than one limit. 4

Or

Discuss the convergence of the sequence $\{S_n\}$, where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$.

(d) Discuss the convergence of the sequence $\{S_n\}$, where $S_n = \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}, \forall n \in N$. 4

Or

Find the limit of $\frac{1+3+5+\cdots+(2n-1)}{n^2}$.

2. (a) Write the necessary condition for convergence of an infinite series Σu_n. 1

(b) Write the statement of comparison test. 2

(c) State the principle of Leibnitz's test of an alternating series. 2

(d) Test the convergence of the following series : 5

$$\frac{1}{2} + \frac{1}{2 \cdot 2^2} + \frac{1}{3 \cdot 2^3} + \frac{1}{4 \cdot 2^4} + \cdots$$

Or

Show that the series $\Sigma \frac{1}{n}$ does not converge.

(e) Prove that a positive term series converges if and only if the sequence of its partial sums is bounded. 5

3. *(a)* Write the transformed equation of $7x^3 + 2x^2 + 3 = 0$ to solve by Cardan's method. 1

(b) Every equation of odd degree has only complex roots. State true or false. 1

(c) Write the nature of the roots of the equation $x^3 + px + q = 0$, if p and q are positive. 2

(d) If α, β, γ are the roots of the equation $6x^3 - 11x^2 - 3x + 2 = 0$, and are in harmonic progression, then find the value of $\gamma\alpha$. 3

(e) Solve the following by Cardan's method : 5

$$x^3 - 6x - 9 = 0$$

Or

If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, find the equation whose roots are $\alpha - \frac{1}{\beta\gamma}$, $\beta - \frac{1}{\alpha\gamma}$, $\gamma - \frac{1}{\alpha\beta}$.

(f) Find the sum of the fourth powers of the roots of $x^3 - x - 1 = 0$. 3

GROUP—B

(Trigonometry)

4. *(a)* Write the value of $\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}$. 1

(b) Write the roots of the equation $x^2 - 2x\cos\theta + 1 = 0$. 1

(c) Find all the values of $(1+i)^{\frac{1}{3}}$. 3

Or

Expand $\cos^3 x$ in powers of x.

(d) If

$$\sin\alpha + \sin\beta + \sin\gamma = \cos\alpha + \cos\beta + \cos\gamma = 0$$

then find the value of

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$$ 3

Or

If $(1+x)^n = P_0 + P_1x + P_2x^2 + \cdots$, then show that $P_0 - P_2 + P_4 - \cdots = 2^{\frac{n}{2}}\cos\frac{n\pi}{4}$.

5. Show that

$$\log(a+ib) = \log\sqrt{a^2+b^2} + i\tan^{-1}\frac{b}{a}$$ 5

Or

Show that $\log i = i\frac{\pi}{2}(4n+1)$.

6. *(a)* Write the value of the series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\cdots$. 1

(b) Show that

$$\frac{1}{1\cdot 3}+\frac{1}{5\cdot 7}+\frac{1}{9\cdot 11}+\cdots=\frac{\pi}{8}$$ 3

Or

Find the sum of the inifinite series $1-\frac{1}{3^2}+\frac{1}{5\cdot 3^2}-\frac{1}{7\cdot 3^3}+\cdots$ up to ∞.

7. *(a)* Choose the correct answer : 1

$\sin h^{-1}x$ is equal to

(i) $i\sin^{-1}(ix)$

(ii) $-i\sin^{-1}(ix)$

(iii) $-\sin^{-1}(ix)$

(iv) $-\sin^{-1}x$

(b) If $\cos^{-1}(x+iy) = A+iB$, then show that $x = \cos A \cosh B$. 2

(c) Obtain the sum of the series $\sin\alpha + \frac{1}{2}\sin 2\alpha + \frac{1}{2^2}\sin 3\alpha + \cdots$. 5

Or

Separate $\sin^{-1}(x+iy)$ into real and imaginary parts.

GROUP—C

(Vector Calculus)

8. (a) Find $\frac{\partial}{\partial x}(x^3\hat{i} + y^2\cos x\hat{j} + \sin^2 x\hat{k})$. 1

(b) Let $\vec{V} = x^2y\hat{i} + 2y^3\hat{j} + 3z^4\hat{k}$. Find $\nabla\cdot\vec{V}$ at (1, 1, 1). 2

(c) Prove that $\nabla\times(\phi\vec{A}) = (\nabla\phi)\times\vec{A} + \phi(\nabla\times\vec{A})$. 5

Or

Prove that $\nabla\cdot\left(\frac{\vec{r}}{r^3}\right) = 0$.

(d) Define directional derivative of a scalar function. 2

(e) Prove that $\nabla f(r) = \frac{f'(r)\vec{r}}{r}$. 5

Or

Prove that

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

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