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5 SEM TDC DSE MTH (CBCS)
1.1/1.2/1.3 (H)

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(Held in January/February, 2022)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE-1

Full Marks : 80

Pass Marks : 32

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

Paper : DSE-1.1

(Analytical Geometry)

1. Answer the following questions :

(a) Write the processes to sketch the parabola. 3

(b) Identify and sketch the curve

$$x = y^2 - 4y + 2$$

and also label the focus, vertex and directrix. 6

- (c) Describe the graph of the curve

$$3(x+2)^2 + 4(y+1)^2 = 12$$

Also find its centre and foci.

6

Or

Describe the graph of the hyperbola

$$16x^2 - y^2 - 32x - 6y - 57 = 0$$

and sketch its graph.

2. Answer the following :

- (a) Fill in the blank :

1

The set of points in the plane, the sum of whose distances from two fixed points is a positive constant greater than the distance between the fixed points is _____.

- (b) Write True or False :

1

A hyperbola is the set of all points in the plane that are equidistant from a fixed line and a fixed point not on the line.

- (c) Suppose that an ellipse has semi-major axis a , semi-minor axis b and foci $(\pm c, 0)$. Then write the expression c in terms of a and b .

1

- (d) Find the equation of the parabola that has its vertex at (1, 2) and focus at (4, 2). Also state the reflection property of parabola. 6
- (e) Find the equation of the ellipse whose length of major axis is 26 and foci ($\pm 5, 0$) and also sketch it. 6

Or

Find and sketch the curve of the hyperbola whose foci (6, 4) and (-4, -4) and eccentricity is 2.

3. Answer the following questions :

- (a) Write the condition that the quadratic equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

represents hyperbola. 1

- (b) Determine a rotation angle θ that will eliminate the xy -term of the conic

$$2x^2 + xy + 2y^2 + x - y = 0 \quad 2$$

- (c) Consider the equation

$$x^2 - xy + y^2 - 6 = 0$$

Rotate the coordinate axes to remove the xy -term. Then identify the type of conic represented by the equation and sketch its graph. 6

(d) Let an $x'y'$ -coordinate system be obtained by rotating an xy -coordinate system through an angle $\theta = 30^\circ$.

(i) Find the $x'y'$ -coordinate of the point whose xy -coordinate is (2, 4).

(ii) Find an equation of the curve

$$2x^2 + 2\sqrt{3}xy = 3$$

in $x'y'$ -coordinate.

6

Or

Identify and sketch the curve

$$153x^2 - 129xy + 97y^2 - 30x - 40y - 200 = 0$$

4. Answer the following questions :

(a) Define sphere.

1

(b) Write the equation of the sphere whose end points of the diameter is given.

1

(c) Find the equation of the sphere whose centre is (2, 3, 1) and radius is 5 units.

4

(d) Find the equation of the sphere through the origin and intersecting coordinate axes at distances a , b and c from the origin.

5

Or

A plane passes through a fixed point (a, b, c) and meets the axes in A, B, C . Show that the locus of the centre of the sphere $OABC$ is

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$

5. Answer the following questions :

(a) Define great circle. 1

(b) Write the condition that the plane

$$ax + by + cz + d = 0$$

be a tangent plane to the sphere

$$x^2 + y^2 + z^2 = r^2$$
1

(c) Find the radius and centre of the circle

$$x^2 + y^2 + z^2 - x - y - z - 1 = 0, \quad x + y + z = 0$$
5

(d) Find the equation of the sphere for which the circle

$$x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$$

and $2x + 3y - 4z = 8$

is a great circle. 5

Or

Find the equation of the tangent planes of the sphere

$$x^2 + y^2 + z^2 - 4x - 4y - 4z + 10 = 0$$

which are parallel to the plane $x - z = 0$.

6. Answer the following questions :

(a) Write the name of cylindrical surface given by the equation

$$\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{12} = 1$$

1

(b) Prove that the two spheres

$$x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$$

and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$

intersect each other orthogonally.

2

(c) Show that the plane

$$2x - 2y + z + 12 = 0$$

touches the sphere

$$x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$$

Also find the point of contact.

4

(7)

(d) Classify and sketch the surface

$$9x^2 + 4y^2 + z^2 = 36$$

5

Or

Classify and sketch the surface

$$x^2 + 2z^2 - 6x - y + 10 = 0$$

Paper : DSE-1.2

(Portfolio Optimization)

1. Answer any *five* of the following as directed :

1×5=5

- (a) Define investment.
- (b) What is portfolio?
- (c) Define risk.
- (d) "Return increases with the increase in risk."

(Write True or False)

- (e) Define risk-free asset.
 - (f) What is diversification?
2. (a) If an investment that costs \$250 and is worth \$350 after being held for two years, find annual holding period return (annual HPR) and annual holding period yield (annual HPY).
- (b) Define expected return of an investment. Calculate the expected rate of return of the following economic scenarios :

1+2=3

Economic Condition	Probability	Rate of Return
Strong economy	0.15	0.20
Weak economy	0.15	-0.20
No major change in economy	0.70	0.10

- (c) Write the measures of risk in terms of variance and standard deviation of the estimated distribution of expected returns. What is the value of variance of risk-free investment? 2+2+1=5
- (d) Describe different types of risk of an investment. 5

Or

Describe the relationship between risk and return.

- (e) Write a short note on mutual fund. 4
 - (f) Describe the investment objectives for 25-year-old investors and 65-year-old investors. 4
3. (a) What is risk aversion? 2
- (b) What are the assumptions of the Markowitz's portfolio theory? 5
- (c) Write the formula for the expected return for a portfolio of investments. Calculate the expected return of portfolio of risky assets given by the table : 1+2=3

Weight (w_j) (percent of portfolio)	Expected Security Returns (R_i)
0.20	0.10
0.30	0.11
0.30	0.12
0.20	0.13

- (d) What are the variance and standard deviation of returns for an individual investment? Calculate the variance for an individual risky asset given by the following table :

2+2+3=7

Possible rate of return (R_j)	Expected Security Return [$E(R_j)$]	Probability (P_i)
0.08	0.103	0.35
0.10	0.103	0.30
0.12	0.103	0.20
0.14	0.103	0.15

Or

Describe variance and standard variation of returns for a portfolio of investments.

7

- (e) Define risk-free portfolio using standard deviation of a portfolio of investments.

2

- (f) Write short notes on any two of the following :

3×2=6

- (i) Optimal portfolio
- (ii) Efficient frontier
- (iii) Portfolio with short sales

4. Answer any *three* of the following questions :

5×3=15

- (a) Write five assumptions of capital market theory.
- (b) Derive the risk-return combination equation of capital market theory.
- (c) Derive the equation of the capital asset pricing model (CAPM).
- (d) Determine the expected rate of return with CAPM for the following five stocks :

<i>Stock</i>	<i>Beta</i>
A	0.70
B	1.00
C	1.15
D	1.40
E	-0.30

where economy's RER = 0.05 and expected return on the market portfolio $E(R_M) = 0.09$.

5. What is security market line (SML)? What are the differences between capital market line (CML) and security market line (SML)? 1+2=3

6. Suppose that during the most recent 10 years period, the average annual total rate of return including dividends on an aggregate market portfolio was 14 percent ($\overline{R}_M = 0.14$) and the average nominal rate of return on government T-bills was 8 percent ($\overline{RFR} = 0.08$). As administrator of a large pension fund that has been divided among three money managers during the past 10 years. Decide by calculating T values whether to renew their investment management contracts based on the following results :

<i>Investment Manager</i>	<i>Average Annual Rate of Return</i>	<i>Beta</i>
W	0.12	0.90
X	0.16	1.05
Y	0.18	1.20

Also plot their portfolios with security market line (SML).

Or

Describe sharp portfolio performance measure with example.

Paper : DSE-1.3

(Financial Mathematics)

UNIT—I

1. Answer the following questions : 1×4=4

- (a) Define cash flow.
- (b) Write to which greater expected return is related.
- (c) If the interest rate is r , then write the price of an investment that pays A after one year.
- (d) Write which entirely determines internal rate of return.

2. Answer the following questions : 2×4=8

- (a) Explain the viewpoint of investment.
- (b) Write about the investment and return for the situation represented by cash flow $(-1, 1.4)$.
- (c) Write the relation between present value and future value.
- (d) Explain callable bond.

3. Answer any *four* of the following questions :

4×4=16

- (a) Explain comparison principle.
- (b) Write the main features of hedging.

- (c) Write the objectives of pure investment.
- (d) Show that growth under compound interest is geometric.
- (e) Compute the future value of the cash flow stream $(-4, 1, 2, 1)$.
- (f) Describe effective interest rate and nominal rate.

4. Answer any *two* of the following questions :

$6 \times 2 = 12$

- (a) Find the internal rate of return by solving the equation $x^3 + x^2 + x = 2$ (use Newton-Raphson method).
- (b) State and prove the main theorem of internal rate of return.
- (c) Describe duration.
- (d) Show that derivation of price P with respect to yield λ of a fixed income security is

$$\frac{dP}{d\lambda} = -D_m P$$

where D_m is modified duration.

UNIT—II

5. Answer the following questions : 1×4=4

- (a) Define random variable.
- (b) Define diversification.
- (c) Write one property of feasible set.
- (d) Write through which capital market line passes.

6. Answer the following questions : 2×4=8

- (a) Write about short selling.
- (b) Find the expected value of the number of spots on a roll of a die.
- (c) Write two properties of expected value.
- (d) Define covariance of two random variables x_1, x_2 .

7. Answer any *four* of the following questions : 7×4=28

- (a) Show that the rate of return acts like an interest rate.
- (b) Find the mathematical expression for total return.

- (c) Show that the variance of the return of the portfolio

$$\sigma^2 = \sum_{i,j=1}^n w_i w_j \sigma_{ij}$$

- (d) State the capital asset pricing model and prove it.
- (e) Define mean standard deviation diagram and show that

$$\text{var}(x) = E(x^2) - \bar{x}^2$$
