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5 SEM TDC MTMH (CBCS) C 11

2021

(Held in January/February, 2022)

MATHEMATICS

(Core)

Paper : C–11

(Multivariate Calculus)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) Define limit of a function of two variables. 1

(b) Find

$$\lim_{(x,\, y)\to(0,\, 1)} \frac{x - xy + 3}{x^2 y + 5xy - y^3}$$ 1

(c) Show that the function

$$f(x, y) = \begin{cases} \dfrac{xy^3}{x^2 + y^6}; & (x, y) \neq (0, 0) \\ 0 \quad ; & (x, y) = (0, 0) \end{cases}$$

is not continuous at $(0, 0)$. 3

(d) If $u = e^{xyz}$, then show that

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$$ 3

Or

If $w = x \sin y + y \sin x + xy$, then verify that $w_{xy} = w_{yx}$.

2. (a) Define total differential of a function of two variables. 1

(b) For changes in a function's values along a helix $w = xy + z$, $x = \cos t$, $y = \sin t$ and $z = t$. Find $\frac{dw}{dt}$. 2

(c) State and prove sufficient condition for differentiability of a function of two variables. 4

Or

Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$, $x = \frac{r}{s}$, $y = r^2 + \log x$ and $z = 2r$.

2+2=4

3. (a) Find the equation of tangent plane at (1, 1, 1) for the curve $x^2 + y^2 + z^2 = 3$. 2

(b) Find the local extreme values of the function

$$f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$$ 3

(c) Find the extreme values of $f(x, y) = xy$ taken on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$ by the method of Lagrange's multipliers. 5

Or

The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin.

4. (a) Find ∇f, if

$$f(x, y, z) = x^2 + y^2 - 2z^2 + z\log x$$ 1

(b) Prove that $\operatorname{div} \vec{r} = 3$. 2

(c) Find curl $\vec{f}$, where

$$\vec{f} = x^2 y\hat{i} + xz\hat{j} + 2yz\hat{k}$$ 2

5. (a) Write one property of double integral. 1

(b) Evaluate

$$\iint_R f(x, y)\, dA$$

for $f(x, y) = 1 - 6x^2 y$, $R : 0 \le x \le 2$ and $-1 \le y \le 1$. 2

(c) Find the area enclosed by the Lemniscate $r^2 = 4\cos 2\theta$. 3

6. (a) Define triple integrals. 2

(b) Evaluate : 2

$$\int_{y=0}^{3} \int_{x=0}^{2} \int_{0}^{1} (x + y + z)\, dz\, dx\, dy$$

(c) Find the volume of the upper region D cut from the solid sphere $\rho \le 1$ by the cone $\phi = \frac{\pi}{3}$. 5

Or

Find the volume of the region enclosed by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 4$.

7. (a) Write the formula for triple integral in cylindrical coordinates. 1

(b) Evaluate : 4

$$\int_0^{2\pi}\int_0^{\theta/2\pi}\int_0^{3+24r^2} dz r\, dr\, d\theta$$

Or

Find the volume of the region in the first octant bounded by the coordinate planes, the plane $y = 1 - x$ and the surface $z = \cos\frac{\pi x}{2}$, $0 \le x \le 1$.

8\. (a) Find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ for the transformation $x = u\cos v$ and $y = u\sin v$. 1

(b) Evaluate

$$\int_0^4\int_{x=\frac{y}{2}}^{x=\frac{y}{2}+1} \frac{2x-y}{2}\, dx\, dy$$

by applying the transformation $u = \frac{2x-y}{2}$, $v = \frac{y}{2}$. 3

(c) Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segment C joining the origin and the point $(1, 1, 1)$. 3

Or

Evaluate $\int_C (xy+y+z)\,ds$ along the curve

$$\vec{r}(t) = 2t\hat{i} + t\hat{j} + (2-2t)\hat{k}, \quad 0 \le t \le 1$$

9. (a) Define vector field and write the formula for vector field in three dimensions. 1+1=2

(b) A coil spring lies along the helix

$$\vec{r}(t) = (\cos 4t)\hat{i} + (\sin 4t)\hat{j} + t\hat{k}; \ 0 \le t \le 2\pi$$

The spring density is a constant $\delta = 1$. Find the spring's mass and moments of inertia and radius of gyration about the z-axis. 4

Or

Find the work done by the force

$$\vec{F} = (y - x^2)\hat{i} + (z - y^2)\hat{j} + (x - z^2)\hat{k}$$

over the curve $\vec{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$; $0 \le t \le 1$ from $(0, 0, 0)$ to $(1, 1, 1)$.

(c) Write the fundamental theorem for line integrals. 2

10. (a) State Green's theorem in flux-divergence form. 1

(b) Evaluate the integral $\oint_C (xy\,dy - y^2 dx)$ by using Green's theorem, where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$. 3

(c) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ by using Stokes' theorem, if $\vec{F} = xz\hat{i} + xy\hat{j} + 3xz\hat{k}$ and C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant and traversed counter-clockwise. 5

Or

Find the surface area of a sphere of radius a with parametrization formula

$$\vec{r}(\phi, \theta) = (a\sin\phi\cos\theta)\hat{i} + (a\sin\phi\sin\theta)\hat{j} + (a\cos\phi)\hat{k}$$

where $0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$.

(d) State and prove divergence theorem. 6

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